# Solution Problem 1

**Eclipses of the Jupiter's Satellite**

a. ( Total Point : 1 ) Assume the orbits of the earth and Jupiter are circles, we can write the centripetal force = equal gravitational attraction of the Sun.

$$G\frac{M_E M_s}{R_E^2} = \frac{M_E V_E^2}{R_E}$$
$$G\frac{M_J M_S}{R_J^2} = \frac{M_J V_J^2}{R_J}$$

(0.5 point)

where

$G$ = universal gravitational constant
$M_S$ = mass of the Sun
$M_E$ = mass of the Earth
$M_J$ = mass of the Jupiter
$R_E$ = radius of the orbit of the Earth
$V_E$ = velocity of the Earth
$V_J$ = velocity of Jupiter

Hence

$$\frac{R_J}{R_E} = \left(\frac{v_E}{v_J}\right)^2$$

We know

$$T_E = \frac{2\pi}{\omega_E} = \frac{2\pi R_E}{v_E}, \text{ and}$$
$$T_J = \frac{2\pi}{\omega_J} = \frac{2\pi R_J}{v_J}$$

we get

$$\frac{T_E}{T_J} = \frac{\frac{R_E}{v_E}}{\frac{R_J}{v_J}} = \left(\frac{R_E}{R_J}\right)^{3/2}$$
$$R_J = 779.8 \times 10^6 \,\text{km}$$

( 0.5 point )

b. **( Total Point: 1 )** The relative angular velocity is

$$\omega = \omega_E - \omega_J = 2\pi\left(\frac{1}{365} - \frac{1}{11.9 \times 365}\right)$$
$$= 0.0157\,\text{rad/day}$$

( 0.5 point )

and the relative velocity is

$$v = \omega R_E = 2.36 \times 10^6\,\text{km/day}$$
$$= 27.3 \times 10^3\,\text{km}$$

( 0.5 point )

c. **( Total Point: 3 )** The distance of Jupiter to the Earth can be written as follows

$$d(t) = R_J - R_E$$
$$d(t).d(t) = (R_J - R_E).(R_J - R_E)$$

(1.0 point)

$$\begin{aligned} d(t) &= \left(R_J^2 + R_E^2 - 2R_E R_J \cos\omega t\right)^{1/2} \\ &\approx R_J \left(1 - 2\left(\frac{R_E}{R_J}\right)\cos\omega t + \ldots\right)^{1/2} \\ &\approx R_J \left(1 - \frac{R_E}{R_J}\cos\omega t + \ldots\right) \end{aligned}$$

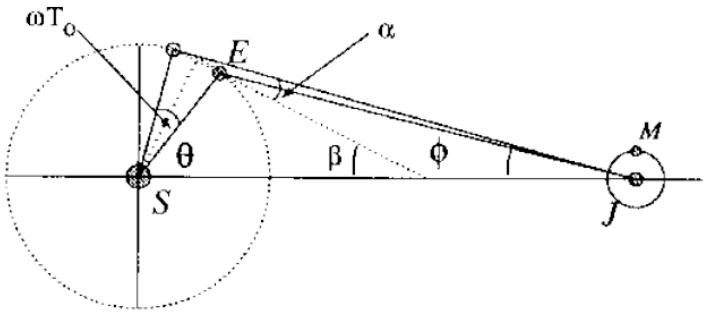



Figure 1: Geometrical relationship to get $\Delta d(t)$

The relative error of the above expression is the order of

$$\left(\frac{R_E}{R_J}\right)^2 \approx 4\%$$

The observer saw M begin to emerge from the shadow when his position was at $d(t)$ and he saw the next emergence when his position was at $d(t + T_0)$/ Light need time to travel the distance $\Delta d = d(t + T_0) - d(t)$ so the observer will get apparent period $T$ instead of the true period $T_o$.

$$\begin{aligned} \Delta d &= R_E \left(\cos\omega t - \cos\omega\left(t + T_0\right)\right) \\ &\approx R_E \omega T_0 \sin\omega t \end{aligned}$$

(1.0 point)

because $\omega T_0 \approx 0.03$, $\sin\omega t + \ldots$, $\cos\omega T_0 \approx 1 - \ldots$

We can also get this approximation directly from the geometrical relationship from Figure 1.

(1.0 point)

or we can use another method.

From the figure above we get

$$\beta = (\phi + \alpha)$$
$$\frac{\omega T_0}{2} + \beta + \theta = \frac{\pi}{2}$$

(1.0 point)

$$\begin{aligned}\Delta d &\approx \omega T_0 R_E \cos\alpha \\ &\approx \omega T_0 R_E \sin\left(\omega t + \frac{\omega T_0}{2} + \phi\right)\end{aligned}$$
$$\omega T_0 \approx 0.03 \text{ and } \phi \approx 0.19$$

(1.0 point)

d. **( Total Point: 2)**

$$T - T_0 \approx \frac{\Delta d(t)}{c};\ c = \text{velocity of light}$$
$$T \approx T_0 + \frac{\Delta d(t)}{c} = T_0 + \frac{R_E \omega T_0 \sin\omega t}{c}$$

(1.0 point)

e. **Total Point : 2 from**

$$T_{\max} = T_0 + \frac{R_E \omega T_0}{c}$$

we get

$$\frac{R_E \omega T_0}{c} = 15$$

Hence

$$C = 2.78 \times 10^5 \,\text{km/s}$$

(1.0 point)