Solution Problem 2

Detection of Alpha Particles

a. From the given range-energy relation and the data supplied we get

$$E = \left(\frac{R\alpha}{0.318}\right)^{2/3} \text{MeV} = \left(\frac{5.50}{0.318}\right)^{2/3} = 6.69\,\text{MeV}$$

(0.5 point)

since $W_{\text{ion-}\rho\text{air}} = 35\,\text{eV}$, then

$$N_{\text{ion-}\rho\text{air}} = \frac{6.69 \times 10^6}{35} = 1.9 \times 10^5$$

(0.5 point)

Size of voltage pulse:

$$\Delta V = \frac{\Delta Q}{C} = \frac{N_{\text{air-}\rho\text{air}}\, e}{C}$$
$$\text{with} \quad C = 45\,\text{pF} = 4.5 \times 10^{-11}$$

(0.5 point)

Hence

$$\Delta V = \frac{1.9 \times 10^5 \times 1.6 \times 10^{-19}}{4.5 \times 10^{-11}}\,\text{V} = 0.68\,\text{mV}$$

(0.5 point)

b. Electrons from the ions-pairs produced by α particles from a radioactive sources of activity A (=number of α particles emitted by the sources per second) which enter the detector with detection efficiency 0.1, will produce a collected current.

$$I = \frac{Q}{t} = 0.1 \times A\, N_{\text{ion-}\rho\text{air}}\, e$$
$$= 0.1 \times A \times 1.9 \times 10^5 \times 1.6 \times 10^{-19}\,\text{A}$$

(1.0 point)

With $I_{\text{min}} = 10^{-12}\,\text{A}$, the

$$A_{\text{min}} = \frac{10^{-12}\,\text{dis}\,\text{s}^{-1}}{1.6 \times 1.9 \times 10^{-15}} = 330\,\text{dis}\,\text{s}^{-1}$$

(1.0 point)

Since $1\,\text{Ci} = 3.7 \times 10^{10}\,\text{dis}\,\text{s}^{-1}$ then

$$A_{\text{min}} = \frac{330}{3.7 \times 10^{10}}\,\text{Ci} = 8.92 \times 10^{-9}\,\text{Ci}$$

(1.0 point)

c. With time constant

$$\tau = RC\,(\text{with } C = 45 \times 10^{-12}\,\text{F}) = 10^{-3}\,\text{s}$$

$$R = \left(\frac{1000}{45}\right)\text{M}\Omega = 22.22\,\text{M}\Omega$$

(0.5 point)

For the voltage signal with height $\Delta V = 0.68\,\text{mV}$ generated at the anode of the ionization chamber by 6.69 MeV α particles in problem (a), to achieve q 0.25 V= 250 mV voltage signal, the necessary gain of the voltage pulse amplifier should be

$$G = \frac{250}{0.68} = 368$$

(0.5 point)

d. By symmetry, the electric field is directed radially and depends only on distance the axis and can be deducted by using Gauss' theorem.

If we construct a Gaussian surface which is a cylinder of radius τ and length l, the charge contained within it is $\sigma\, l$.

The surface integral

$$\int E \cdot dS = 2\pi r l E$$

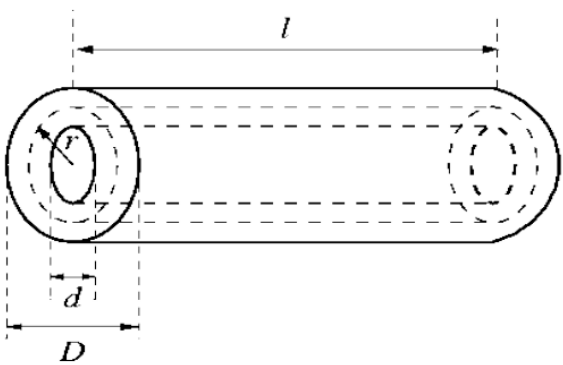


Figure 1 : The Gaussian surface used to calculate the electric field E.

(1.0 point)

Since the field E is everywhere constant and normal to the curved surface. By Gauss's theorem:

$$2\pi r l E = \frac{\lambda l}{\varepsilon_0}$$

so

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Since E is radial and varies only with τ, then $E = -\dfrac{dV}{dr}$ and the potential V can be found by integrating $E(\tau)$ with respect to τ, if we call the potential of inner wire V_0, we have

$$V(r) - V_0 = -\frac{\lambda}{2\pi\varepsilon_0}\int_{d/2}^{\tau}\frac{dr}{r}$$

Thus

$$V(r) - V_0 = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{2r}{d}\right)$$

(1.0 point)

We can use this expression to evaluate the voltage between the capacitor's conductors by setting $r = \dfrac{D}{2}$, giving a potential difference of

$$V = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{D}{d}\right)$$

since the charge Q in the capacitor is $\sigma\, l$, and the capacitance C is defined by $Q = CV$, the capacitance per unit length is

$$\frac{2\pi\varepsilon L_0}{\ln\frac{D}{d}}$$

(1.0 point)

The maximum electric field occurs where r minimum, i.e. at $r = \dfrac{d}{2}$. if we set the field at $r = \dfrac{d}{2}$ equal to the breakdown field E_b, our expression for E® shows that the charges per unit length σ in the capacitor must be $E_b\pi_0 d$. Substituting for the potential difference V across the capacitor gives

$$V = \frac{1}{2}E_b d\ln\left(\frac{D}{d}\right)$$

Taking $E_b = 3\times10^6\,\mathrm{V}$, $d = 1\,\mathrm{mm}$, and $D = 1\,\mathrm{cm}$, gives $V = 3.453.45\,\mathrm{kV}$.

(1.0 point).