Thermal Vibration of Surface Atoms

(1) (a) The wavelength of the incident electron is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.60 \times 10^{-19} \times 64.0}}$$

$$= 1.53 \times 10^{-10}\,\text{m} = 1.53\,\text{Å}$$

(b) Consider the interference between the atomic rows on the surface as shown in Fig. 3c.

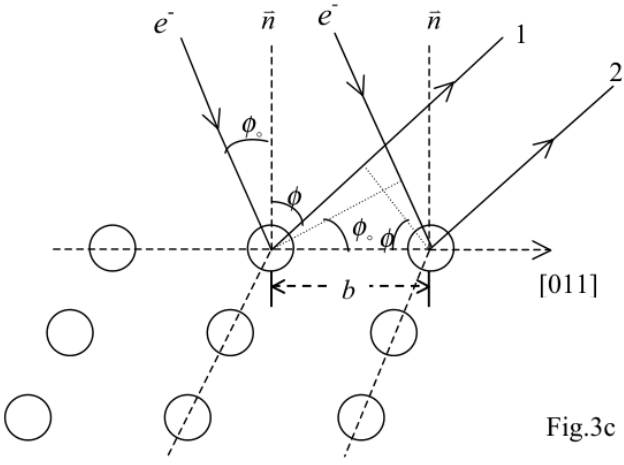


Fig.3c

The path difference between electron beam 1 and 2 is

$$\Delta\ell = b(\sin\phi - \sin\phi_\circ) = n\lambda$$

Given $\phi_\circ = 15.0°$, $\lambda = 1.53\,\text{Å}$ and $b = \frac{a}{\sqrt{2}} = \frac{3.92}{\sqrt{2}} = 2.77\,\text{Å}$, two solutions are possible.

(i) When $n = 0$, $\phi = \phi_\circ = 15.0°$ (Answer 1)

(ii) When $n = 1$

$$\Delta\ell = 2.77(\sin\phi - \sin 15°) = 1 \times 1.53$$

$$\sin\phi = \frac{1.53 + 0.72}{2.77} = 0.812$$

$\phi = 54.3°$ (Answer 2)

For $n = 2$, no solution exists as $\Delta\ell = 2.77(\sin\phi - \sin 15°) = 2 \times 1.53$ and $\sin\phi > 1$.

(2) $I = I_0 \exp\langle -(\vec{u} \cdot \Delta\vec{K})^2 \rangle$

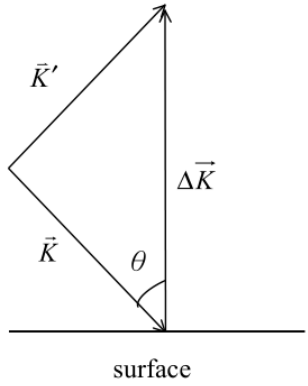


Fig. 3d

For the specularly reflected beam, we have from Fig. 3d

$$\Delta\vec{K} = \vec{K}' - \vec{K} = 2K\cos\theta\,\hat{x}$$

where $\hat{x}$ is the unit vector in the direction of the surface normal. Take the x-component of $\vec{u}$, we then obtain

$$I = I_0 e^{-\langle u_x^2(t)\cdot 4K^2\cos^2\theta\rangle} = I_0 e^{-4K^2\cos^2\theta\langle u_x^2(t)\rangle} \tag{2}$$

The vibration in the direction of the surface normal of the surface atoms is simple harmonic, take

$$u_x(t) = A\cos\omega t$$

$$\langle u_x^2(t)\rangle = \frac{1}{\tau}\int_0^\tau u^2\,dt = \frac{1}{\tau}\int_0^\tau A^2\cos^2\omega t\,dt = \frac{A^2}{\tau}\cdot\frac{\tau}{2} = \frac{A^2}{2}$$

$$\therefore \quad A^2 = 2\langle u_x^2(t)\rangle$$

The total energy E is thus given by

$$E = \frac{1}{2}CA^2 = \frac{1}{2}C\cdot 2\langle u_x^2(t)\rangle = C\langle u_x^2(t)\rangle = m'\omega^2\langle u_x^2(t)\rangle$$

Therefore, one obtains

$$\langle u_x^2(t)\rangle = E/(m'\omega^2)$$

$$E = m'\omega^2\langle u_x^2\rangle = k_BT$$

where m' is the mass of the atom. From either of the above two equations, one then has the following equality

$$\langle u_x^2\rangle = \frac{k_BT}{m'\omega^2} = \frac{k_BT}{m'4\pi^2f^2} \tag{3}$$

From eq. (3) and eq. (2), one obtains

$$I = I_0 e^{-4K^2\cos^2\theta\,\frac{k_BT}{m'4\pi^2f^2}}$$

where $K = \dfrac{2\pi p}{h} = \dfrac{2\pi}{\lambda}$. Accordingly,

$$I = I_0 e^{-\frac{4k_B\cos^2\theta}{m'f^2\lambda^2}T} = I_0 e^{-M'T} \tag{4}$$

and

$$\ln\frac{I}{I_0} = -M'T$$

From the plot of $\ln\frac{I}{I_0}$ versus T, one obtains the slope

$$M' = \frac{4k_B\cos^2\theta}{m'f^2\lambda^2} \tag{5}$$

The slope of the curve can be estimated from Fig. 3b and leads to the result

$$M' = 2.3\times10^{-3}.$$

Using the following data in Eq.(5),

$$k_B = 1.38\times10^{-23}\,\mathrm{J/K}$$

$$\lambda = 1.53\times10^{-10}\,\mathrm{m}$$

$$m' = 195.1\times10^{-3}/(6.02\times10^{23}) = 3.24\times10^{-25}\,\mathrm{kg/atom}$$

one finds

$$2.3 \times 10^{-3} = \frac{4 \times 1.38 \times 10^{-23} \cdot \cos^2 15^\circ}{\dfrac{195.1 \times 10^{-3}}{6.02 \times 10^{23}} \times f^2 \times (1.53 \times 10^{-10})^2}$$

The solution for frequency is then

$$f^2 = 3.0 \times 10^{24}\,(\text{new}) \Rightarrow f = 1.7 \times 10^{12}\,\text{Hz}$$

Answer (a)

From $\langle u_x^2 \rangle = \dfrac{k_B T}{m' 4\pi^2 f^2}$, $T = 300\,\text{K}$, one finally obtains

$$\langle u_x^2 \rangle = \frac{1.38 \times 10^{-23} \cdot 300}{\dfrac{195.1 \times 10^{-3}}{6.02 \times 10^{23}} \times 4\pi^2 \times 3.0 \times 10^{24}} = 1.1 \times 10^{-22}\,\text{m}^2\,(\text{new})$$

and

$$\sqrt{\langle u_x^2 \rangle} = 1.0 \times 10^{-11}\,\text{m} = 0.10\,\text{Å}\,(\text{new})$$

Ans