Solution and Marking Scheme

Theory

III. Plasma Lens

a) Consider cylindrical Gaussian surface of radius r and length ℓ about the central axis.

From Gauss's law

$$E\,2\pi r\ell = \frac{1}{\varepsilon_0} \quad \text{(charge inside)} \qquad (0.5 \text{ points})$$

$$= -\frac{1}{\varepsilon_0}\, ne(\pi r^2 \ell)$$

$$E_r = -\frac{ner}{2\varepsilon_0} \qquad (0.5 \text{ point})$$

b) From Ampere's law, $\quad B_\theta = -\dfrac{\mu_0 ner v}{2} \qquad (2 \text{ points})$

c) The net Lorentz force is

$$\vec{F} = \left(\frac{ne^2 r}{2\varepsilon_0} - \frac{\mu_0 ne^2 r v^2}{2}\right)\hat{r} = \frac{ne^2 r}{2\varepsilon_0}\left(1 - \frac{v^2}{c^2}\right)\hat{r}$$

where $c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad (1 \text{ point})$

d) $F_r \to 0$ as $v \to c$, this implies the electric force and magnetic force cancel each other out. (1 point)

e) The stationary plasma particles have $v = 0$, hence $F_{r'} = \pm e E_{r'}$

$$\text{where} \quad E_{r'} = -\frac{neR^2}{2\varepsilon_o r'} + \frac{n_o e r'}{2\varepsilon_o}$$

$$\text{for positive ion} \quad F_{r'} = -\frac{ne^2 R^2}{2\varepsilon_o r'} + \frac{n_o e^2 r'}{2\varepsilon_o}$$

$$\text{for electron} \quad F_{r'} = \frac{ne^2 R^2}{2\varepsilon_o r'} - \frac{n_o e^2 r'}{2\varepsilon_o}$$

and there is no cancellation from the magnetic force.

As a result the plasma electrons will be blown out, and the ions are pulled in. (2 points)

f) The net force on the electron beam in plasma medium is given by,

$$\vec{F} = \frac{ne^2 r}{2\varepsilon_0}\left(1 - \frac{v^2}{c^2}\right)\hat{r} - \frac{n_0 e^2 r}{2\varepsilon_0}\hat{r} \qquad (2 \text{ points})$$

in the limit $v \to c$, $\quad \vec{F} \approx -\dfrac{n_0 e^2 r}{2\varepsilon_0}\hat{r} \qquad (1 \text{ point})$