Question 3 Light deflection by a moving mirror

Reflection of light by a relativistically moving mirror is not theoretically new. Einstein discussed the possibility or worked out the process using the Lorentz transformation to get the reflection formula due to a mirror moving with a velocity $\vec{v}$. This formula, however, could also be derived by using a relatively simpler method. Consider the reflection process as shown in Fig. 3.1, where a plane mirror M moves with a velocity $\vec{v} = v\,\hat{e}_x$ (where $\hat{e}_x$ is a unit vector in the x-direction) observed from the lab frame F. The mirror forms an angle ϕ with respect to the velocity (note that $\phi \leq 90^0$, see figure 3.1). The plane of the mirror has **n** as its normal. The light beam has an incident angle α and reflection angle β which are the angles between $\vec{n}$ and the incident beam 1 and reflection beam 1′, respectively in the laboratory frame F. It can be shown that,

$$\sin\alpha - \sin\beta = \frac{v}{c}\,\sin\phi\,\sin(\alpha+\beta) \tag{1}$$

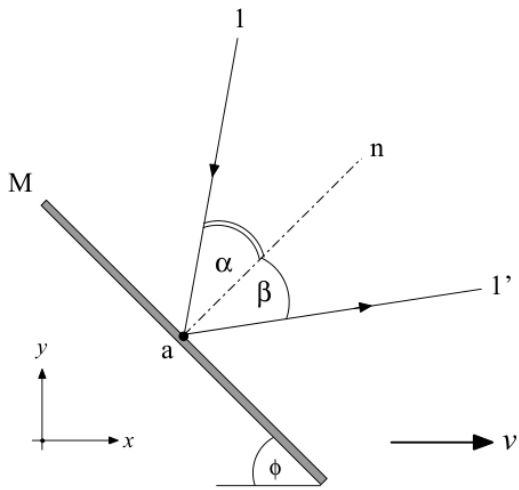


Figure 3.1. Reflection of light by a relativistically moving mirror

3A. Einstein's mirror (2.5 points)

About a century ago Einstein derived the law of reflection of an electromagnetic wave by a mirror moving with a constant velocity $\vec{v} = -v\,\hat{e}_x$ (see Fig. 3.2). By applying the Lorentz transformation to the result obtained in the rest frame of the mirror, Einstein found that:

$$\cos\beta = \frac{\left(1+\left(\frac{v}{c}\right)^2\right)\cos\alpha - 2\,\frac{v}{c}}{1 - 2\,\frac{v}{c}\cos\alpha + \left(\frac{v}{c}\right)^2} \tag{2}$$

Derive this formula using Equation (1) without Lorentz transformation!



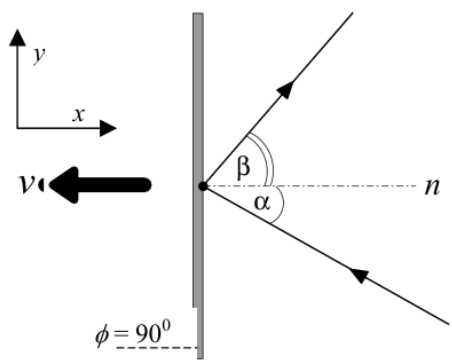


Figure 3.2. Einstein mirror moving to the left with a velocity v.

3B. Frequency shift (2 points)

In the same situation as in 3A, if the incident light is a monochromatic beam hitting M with a frequency f, find the new frequency f' after it is reflected from the surface of the moving mirror. If $\alpha = 30^0$ and $v = 0.6\,c$ in figure 3.2, find frequency shift Δf in percentage of f.

3C. Moving mirror equation (5.5 points)

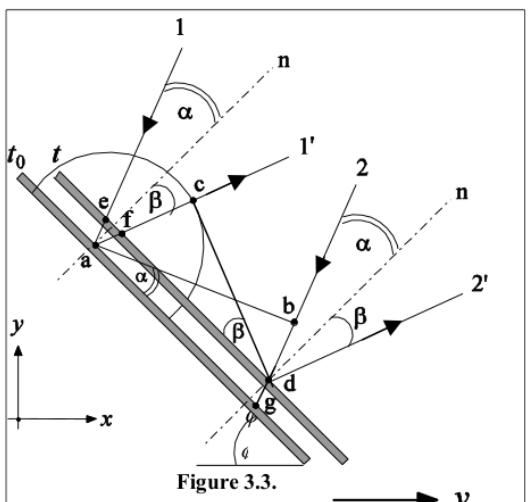


Figure 3.3 shows the positions of the mirror at time t_0 and t. Since the observer is moving to the left, the mirror moves relatively to the right. Light beam 1 falls on point a at t_0 and is reflected as beam $1'$. Light beam 2 falls on point d at t and is reflected as beam $2'$. Therefore, $\overline{ab}$ is the wave front of the incoming light at time t_0. The atoms at point are disturbed by the incident wave front $\overline{ab}$ and begin to radiate a wavelet. The disturbance due to the wave front $\overline{ab}$ stops at time t when the wavefront strikes point d. The semicircle in the figure represents wave-front of the wavelet at time t.

By referring to figure 3.3 for light wave propagation or using other methods, derive equation (1).