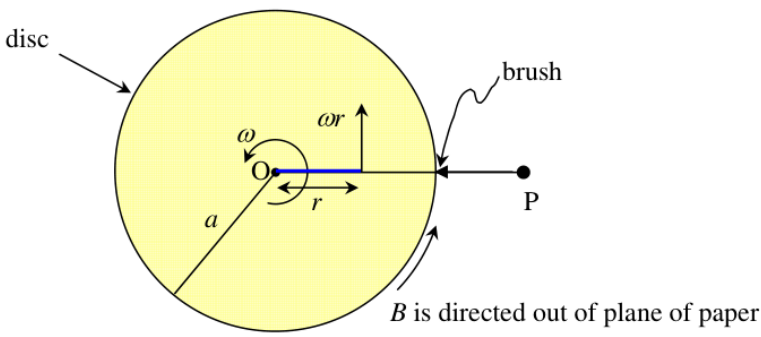
A Self-excited Magnetic Dynamo

2.1) $$L\frac{d}{dt}i + Ri \quad = \quad \mathcal{E} \qquad \text{.....(i)}$$ (1.0 point)

2.2) If the length ℓ of solenoid is much greater than its diameter, then the magnetic flux density in the inside mid-section of the solenoid is given by

$$B \quad = \quad \frac{\mu_0 N i}{\ell} \qquad \text{.....(ii)}$$ (1.5 points)

2.3)



The electric field intensity (E) at distance r from the centre of the axle is

$$E = B\omega r$$

pointing towards the rim of the disc. The induced e.m.f. ($\mathcal{E}$) between terminals P and Q is given by

$$\mathcal{E} \quad = \quad \int_{r=0}^{a} B\omega r dr \quad = \quad \frac{1}{2}B\omega a^2 \quad = \quad \frac{\mu_0 N i \omega a^2}{2\ell}$$

.....(iii) (2.0 points)

2.4) By combining the results in 2.1) and 2.3) we get

$$L\frac{d}{dt}i + Ri \quad = \quad \left(\frac{\mu_0 N a^2 \omega}{2\ell}\right) i$$

$$\frac{d}{dt}i \quad = \quad +\frac{1}{L}\left(\frac{\mu_0 N a^2 \omega}{2\ell} - R\right) i \quad \equiv \quad +\gamma i$$ (0.5 point)

$$\therefore \qquad i\,(t) \quad = \quad i\,(0)\, e^{\gamma t} \qquad \text{.....(iv)}$$ (1.0 point)

$$\text{where} \qquad \gamma \quad \equiv \quad \frac{1}{L}\left(\frac{\mu_0 N a^2 \omega}{2\ell} - R\right)$$

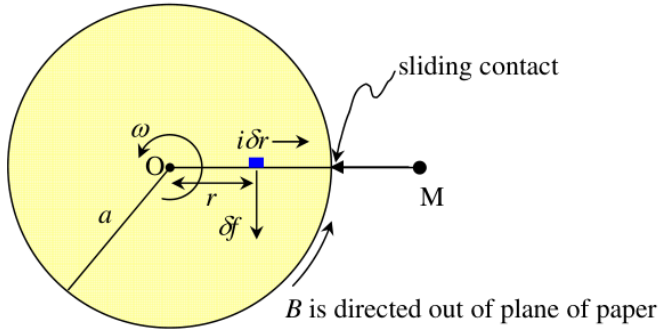
2.5) In order that the current $i\,(t)$ will grow, the value of γ must be positive otherwise the current will gradually decay.

$$\frac{\mu_0 N a^2 \omega}{2\ell} - R \quad \geq \quad 0$$ (1.5 points)

$$\omega_{\min} \quad = \quad \frac{2\ell R}{\mu_0 N a^2} \qquad \text{.....(v)}$$ (0.5 point)

2.6)

Method 1



At the instant t the current is given in 4) as $i\,(t) \;=\; i\,(0)\,e^{\gamma t}$.

The magnetic force δf on the current element $i\delta r$ is $\delta f = Bi\delta r$. (1.0 point)

The torque $\delta\tau = r\delta f = Bir\delta r$ opposes the rotation of the disc.

$$\text{The total torque} \qquad \tau = \int_{r=0}^{r=a} Birdr \quad = \quad \frac{1}{2}Bia^2$$

$$\tau \quad = \quad \frac{\mu_0 N a^2}{2\ell} i^2 \quad = \quad \frac{\mu_0 N a^2}{2\ell} i^2\,(0)\,e^{+2\gamma t} \qquad \text{……(vi)} \qquad \text{(1.0 point)}$$

In order to maintain the angular velocity of the disc at a steady value we must apply a turning torque of equal magnitude and of opposite direction to that of (vi).

Method 2

$$\tau\omega = i^2 R + \frac{d}{dt}\,(\text{magnetic energy in solenoid}) \qquad \text{(1.0 point)}$$

$$= i^2 R + L\frac{di}{dt}i$$

$$\tau = \frac{\mu_0 N a^2}{2\ell} i^2 \quad = \quad \frac{\mu_0 N a^2}{2\ell} i^2\,(0)\,e^{+2\gamma t} \qquad \text{(1.0 point)}$$

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