RF reflectometry for spin readout for silicon quantum computing

Introduction

Developing the idea of quantum computing into a practical technology is one of the largest outstanding challenges in science and technology. A promising path is to manipulate individual electrons in silicon transistors by time-dependent electromagnetic fields.

In this question, we investigate the use of radio frequency (RF) reflectometry and single-electron transistors to read out the state of quantum bits in silicon-based quantum computer prototypes.

Part A and Part B discuss radio wave transmission through cables and transmission lines, part C is devoted to conditions for wave reflection, part D introduces the single-electron transistor, and parts E and F introduce and ask you to optimise the method of reflectometry.

Part A: Lumped element model of a co-axial transmission line (2.0 points)

When modelling DC or low frequency signals, one often assumes that a voltage pulse travels instantaneously throughout the circuit. This assumption is valid when the wavelength of such signals is much longer than the size of the circuit, however when working with radio frequency signals, the dynamics are more complex, and we need to account for the intrinsic capacitance and inductance of our cables in our model. We model a co-axial transmission line which acts as a waveguide as described below, ignoring the small resistance of the copper and the small conductance through the dielectric. Throughout the problem, we consider the large-wavelength limit of electromagnetic waves in the co-axial cable such that electric and magnetic fields are perpendicular to the axis of the cable everywhere (the so-called transverse electromagnetic mode).

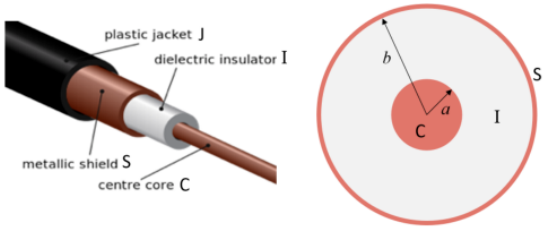


Diagram of a coaxial cable showing C - the centre core, I - the dielectric insulator, S - the metallic shield and J - the plastic jacket.

Consider a co-axial cable consisting of a copper inner core of negligible resistance, negligible magnetic permeability and radius a, covered by an outer co-axial copper shield with inner radius b. A dielectric of dimensionless relative permittivity ε_r and dimensionless relative permeability μ_r separates the layers. When electromagnetic signals propagate through the co-axial cable, they are confined between the inner core and outer shielding.

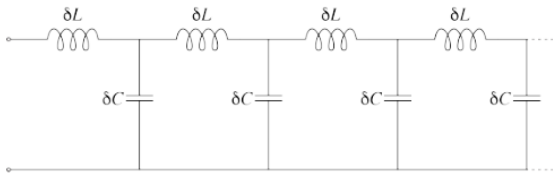
A.1 At what speed do electromagnetic waves propagate in the co-axial cable? 0.2pt

A.2 If there is a charge Δq on a length Δx of the inner core of the co-axial cable, and the outer shield is grounded, find the electric field in the region between the inner core and the shield. 0.2pt

A.3 Find the capacitance per unit length, C_x, of the co-axial cable. You may wish to consider a length Δx of the cable. 0.3pt

A.4 Find the inductance per unit length, L_x, of the cable. 0.3pt

A *lumped element* model of the cable is constructed by considering the inductance and capacitance of short sections of the cable. The inductance is assumed to be a property of the inner core, and the capacitance links the core with the shielding. A diagram of the lumped element model is shown below.



Circuit diagram of lumped element model of coaxial cable.

A.5 i. Show that the impedance Z_0 of a semi-infinite length of cable is $Z_0 = \sqrt{L_x/C_x}$. ii. Find b/a if the cable has impedance $Z_0 = 50\ \Omega$ and is made using a dielectric material with $\varepsilon_\mathrm{r} = 4.0$ and $\mu_\mathrm{r} = 1.0$. 1.0pt

Part B: Hypothetical transmission line with return along a grounded plane (1.0 points)

An alternative hypothetical transmission line is shown in the diagram below. The input signal is sent through a very thin conductor of radius a, which is a distance $d \gg a$ from a highly conductive grounded plane. The material surrounding the conductor has dimensionless relative permittivity ε_r and dimensionless relative permeability μ_r. The return current flows along the grounded plane.

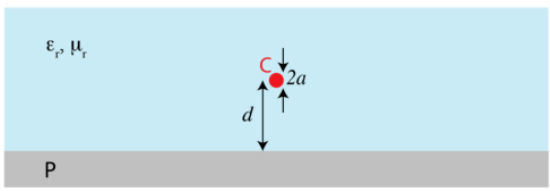


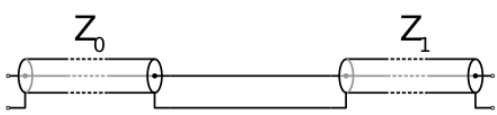
Diagram of a hypothetical transmission line showing C - the conductor of radius a, at a distance $d \gg a$ from P - the grounded conducting plane. The conductor is embedded in a material with dimensionless relative permeability ε_r and dimensionless relative permittivity μ_r.

B.1 Find an expression for the characteristic impedance of this hypothetical transmission line. 1.0pt

Part C: Basics of RF reflectometry (1.2 points)

An electromagnetic wave can propagate in a transmission line in two opposite directions. For each direction of propagation, the characteristic impedance Z_0can be used to relate the voltage V_0 and current I_0amplitudes as in the Ohm's law, $Z_0 = V_0/I_0$.

Consider an interface between two transmission lines, with characteristic impedances Z_0 and Z_1. A schematic diagram of the circuit is shown below.



Circuit diagram of a transmission line of impedance Z_0 connected to a transmission line of impedance Z_1. The physical size of the interface is much smaller than the wavelength.

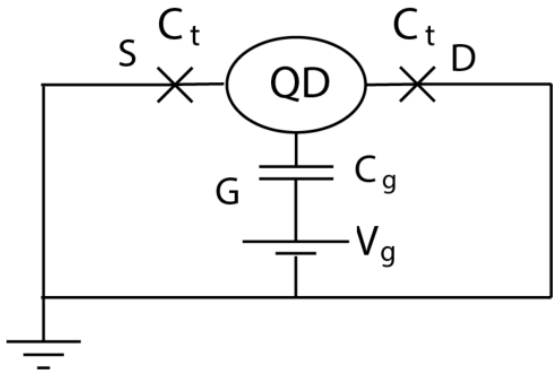
When a signal V_i sent into the transmission line with impedance Z_0reaches the interface it is partially transmitted into the second transmission line, resulting in a signal V_t in that line which propagates forward. Some of the signal may also be reflected, resulting in a backward propagating signal in the initial transmission line V_r.

C.1 Find the reflectance of the interface $\Gamma = V_\mathrm{r}/V_\mathrm{i}$. 1.0pt

C.2 State the condition(s) for the signal V_i to have gained a π phase change on reflection. 0.2pt

Part D: The single electron transistor (3.3 points)

A single electron transistor (SET) consists of a quantum dot, which is a small isolated conductor where electrons can be localised, and of several electrodes in its vicinity. The gate electrode couples capacitatively to the quantum dot, while the two other electrodes --- the source and the drain --- are connected via tunnel junctions, through which electrons can tunnel due to quantum mechanics. A simplified circuit diagram for an SET is shown in the figure.



Circuit diagram representation of an SET. QD is the quantum dot, S is the source, D is the drain and G is the gate.

The capacitance of the gate is C_g and the capacitance of the tunnel junctions is $C_t \ll C_g$. Consider C_g to be the total capacitance of the quantum dot. In this part of the problem, the source and the drain are held at zero potential, and the voltage on the gate electrode is fixed at V_g.

D.1 Consider a state of the SET in which the quantum dot contains n electrons.
i. Find the electrical potential φ_n on the QD.
ii. Find the amount of energy ΔE_n that is necessary to bring an additional electron from the source or the drain onto the QD. 1.5pt

If $\Delta E_n < 0$ then electrons will spontaneously tunnel into the quantum dot until such a number $\mathcal{N} > n$ is reached that $\Delta E_\mathcal{N} \geq 0$. The equilibrium number of electrons $\mathcal{N}$ and the corresponding addition energy $\Delta E_\mathcal{N}$ can be controlled by choosing the appropriate voltage V_g.

D.2 Find an expression for the maximal possible value $E_c = \max \Delta E_\mathcal{N}(V_g)$ of the equilibrium addition energy that can be achieved by tuning the gate voltage of the SET. 0.5pt

If $\Delta E_\mathcal{N} = 0$ then tunnelling of electrons does not require extra energy and SET is in a highly conductive ON state. If $\Delta E_\mathcal{N} > 0$, then the conductance of the SET is reduced (high-resistance OFF).

For the number of electrons on the quantum dot to remain well-defined, certain conditions need to be satisfied. Firstly, if electrons in the source or drain have thermal energies sufficient to move spontaneously onto the quantum dot, the contrast between the ON and OFF states will disappear.

D.3 Find a condition on the temperature of the electrons so that electrons cannot move onto the quantum dot by thermal excitation. 0.5pt

Secondly, tunnelling of electrons onto or off the dot limits the lifetime of their energy states. This tunnelling can be modelled using an effective resistance of the tunnel junction with the characteristic tunnelling time equal to the characteristic time for charging or discharging the quantum dot through the junction.

D.4 i. Estimate the tunnelling time for a quantum dot in terms of capacitance C_t and effective resistance R_t of the tunnel junction.
ii. Find a condition on the effective resistance R_t so that the electrons in the quantum dot retain sufficiently well-defined energy for the ON and OFF states to remain distinct. 0.8pt

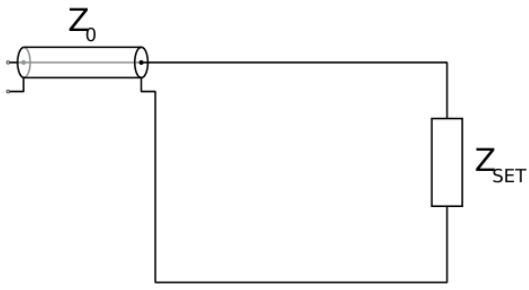
Part E: RF reflectometry to read out SET state (1.0 points)

The state of the SET is sensitive to electrical potentials created by nearby elements of the quantum circuit (such as quantum bits), and distinguishing between ON and OFF states provides a way to read out the information produced by the quantum computer. The SET in the ON state can be modelled by a resistance $R_{\mathrm{ON}} = 100\,\mathrm{k\Omega}$ while in the OFF state we can assume the SET to be a complete insulator (neglecting any capacitative connection between the source and the drain via the SET). While it is possible to determine the state of the SET by measuring the response to an input signal through the source, it is faster to do so using RF reflectometry to measure both the amplitude and phase of the reflected signal, i.e. determined the reflectance Γ.

The change in reflectance due to switching of an SET between ON and OFF states is

$$\Delta\Gamma = |\Gamma_{\mathrm{ON}} - \Gamma_{\mathrm{OFF}}| \ , \tag{1}$$

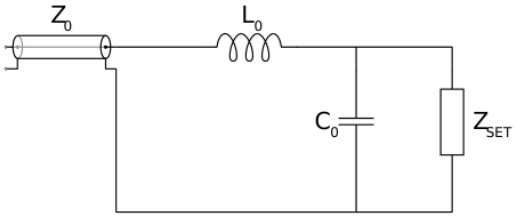
where Γ_{ON} and Γ_{OFF} are the reflectances in two different states.



Circuit diagram of transmission cable of impedance Z_0 connected to an SET.

E.1 Find the change in reflectance $\Delta\Gamma$ between the conductive and insulating states for a typical SET connected to a co-axial cable with impedance of 50 Ω. 0.2pt

In order to increase the change in reflectance, and hence the sensitivity of the RF reflectometry, the circuit is modified by inclusion of an inductor. The intrinsic capacitance due to the device geometry $C_0 \approx 0.4$ pF is also taken into account. The RF reflectometry is conducted using a signal of angular frequency ω_{rf}.



Modified SET circuit.

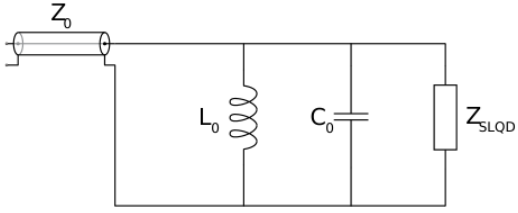
E.2 Estimate the value of the inductance L_0 that can result in the change in reflection on the order of one. Calculate your estimate for L_0 numerically for $\omega_{\mathrm{rf}}/(2\pi) = 100\,\mathrm{MHz}$ and compute the corresponding $\Delta\Gamma$. 0.8pt

Part F: Charge sensing with a single lead quantum dot (1.5 points)

For a scalable quantum computing architecture, the number of wires reaching each individual quantum bit need to be minimized. A promising alternative to an SET for

charge sensing in silicon quantum computing is a Single Lead Quantum Dot (SLQD). In many ways it is similar to an SET, but does not have the source and drain leads. The gate is the only electrode, through which the electron energy states of the quantum dot are controlled and also through which RF reflectometry is conducted.

Like an SET, a SLQD has an OFF in which the SLQD behaves as a total insulator. In contrast to an SET, the ON state of the SLQD is capacitive, with capacitance C_{q}. In order to maximize the difference in reflectance $\Delta\Gamma$ of the SLQD, the following circuit is constructed. The parasitic capacitance $C_0 \approx 0.4$ pF is fixed by circuit geometry, but the value of L_0 and the operating frequency can be changed to optimize the performance. The characteristic impedance of the transmission line is $Z_0 = 50\,\Omega$.



Circuit diagram of the SLQD readout circuit connected to the transmission line.

F.1 Suggest ω_{rf} and $Z_{\mathrm{C}} = \sqrt{L_0/C_0}$ that allow $\Delta\Gamma \sim 1$ for given C_0 and C_q. 1.0pt

Optimal values of L_0 are relatively large and not always technically feasible. Hence, other types of circuit elements may be needed to improve sensitivity of the reflectometry readout circuit.

F.2 Assume that L_0 (and hence Z_C) is fixed. Draw a circuit diagram showing where to place an additional element in the SLQD readout circuit and specify the parameter(s) of this element such that $\Delta\Gamma \sim 1$ can still be achieved without requiring a large inductance. 0.5pt