Acoustic blackbox[1]

A sound source is moving on a circle of fixed radius R in x-y plane with a constant angular speed ω. The circle is also moving with a uniform velocity v_S which makes an angle β ($0° < \beta < 90°$) with the x-axis. Figure 1 shows the trajectory of the source in red color dash-dotted line. At time $t = 0$, the center of the circle is at C, with coordinates (X_C, Y_C), the position of the source is given by vector $\vec{CA}$ from C. In addition, the source S starts moving anticlockwise (counterclockwise) direction (see Fig. 1) simultaneously emitting a sound of frequency f_0 at time $t = 0$. Vector $\vec{CA}$ makes an angle ϕ with respect to the x-axis at $t = 0$ ($\angle BCA = \phi$ in Fig. 1(a)). A detector D is kept in the same plane (x-y) such that its distance from the origin and the angle which the position vector makes with the x-axis are r_D and θ respectively (in Fig. 1(b)). Line joining the detector D and the source S makes an angle α with the x-axis. All the motions of the source and the detector are in the x-y plane only. The speed of sound, c is 330m/s. Also, the net speed of the source is smaller than c.

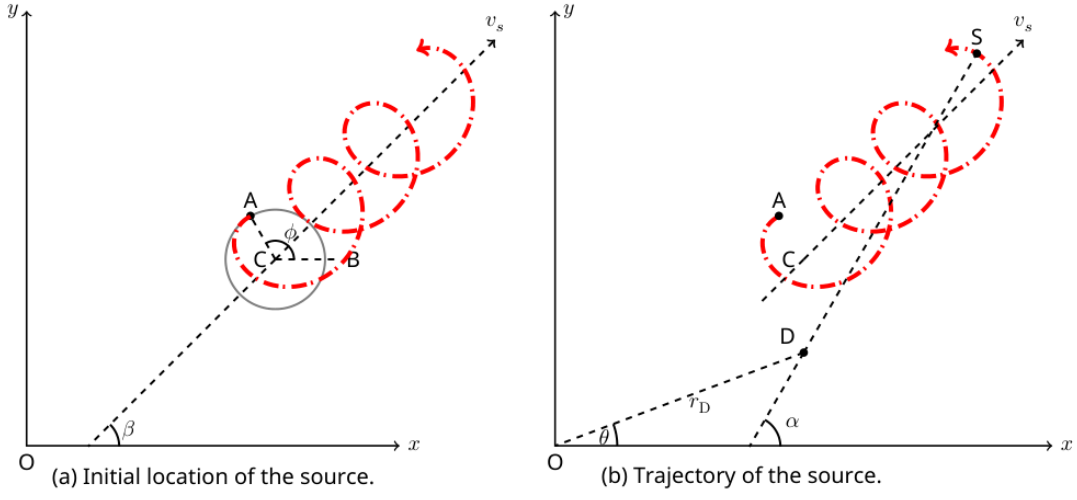


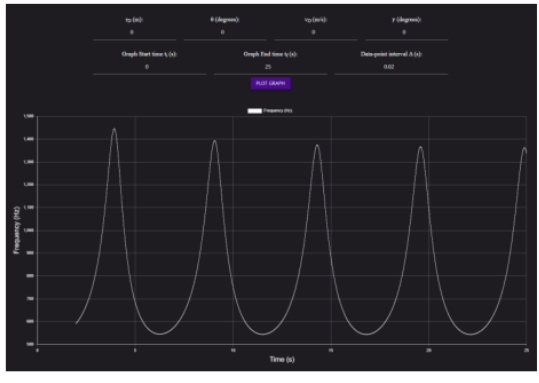
Figure 1

The sound emitted by the source is detected by the detector at time t with frequency $f(t)$. The simulation displays the detected frequency $f(t)$.

A.1 Obtain the equation of the trajectory $(x(t), y(t))$ of the source in terms of β, ϕ, v_s, and the other relevant quantities. 0.2pt

Click on "EQ2- Acoustic Black box" on the exam portal for the experimental exam.

About simulation:
There are two parts to the simulation. The upper part of the panel is the input panel for the detector's parameters and the lower part of the panel is the display of the graphical output of the frequency ($f(t)$) detected by the detector at time t.



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Figure 2:Default simulation screen

You can change the detector's position by entering the values of r_D and θ in the input panel. If you change the value in the panel, you need to click the "PLOT GRAPH" to view the changed output of $f(t)$. You can change the interval at which data is collected in the field "Data-point interval Δ". Note that this interval can only be a multiple of 0.001s.
You can set the detector in uniform motion by entering the speed v_d and the angle γ, which velocity vector $\vec{v}_d$ makes with the x-axis. Again you have to click the "PLOT GRAPH" button to view the output for the changed parameters.
Once the PLOT GRAPH button is clicked, a graph will be displayed for the interval "Graph Start time (t_i)" to "Graph End time (t_f)". In Fig. (2), the panel displays the graph for 0 to 25 s.

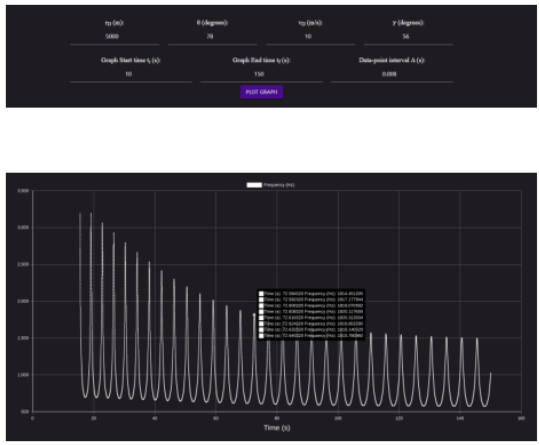


Figure 3: Simulation screen for the changed input values

Graph Start and End time input can also be used to zoom in on a portion of the graph. To illustrate, Fig. (3) shows the output for the case when the detector is placed at 5000 m at angle 78°. The graph output is plotted between 10-150 s. Data is collected at every 0.008 s. When you place the cursor over the curve's data point, it shows the values of the data points. Since the time interval is very small, placing the cursor over the curve may show the values of multiple data points. In Fig. (3), eight data points are shown. **Caution:** If you keep the data point interval (Δ) very small (say 0.001) and plot it for a long time interval (say Graph Start time($t_i = 0$) to Graph End time($t_f = 25s$)), simulation will take a long time and may freeze depending on the capacity of your machine. If you are stuck in such a situation, follow the steps given in the General Instructions. Very small Δ can be used only for a small time interval of the plot. While running simulation maximum number of data points that simulator can plot is 25000, irrespective of Graph End time (t_f (s)).

Try various values of the detector parameters in the simulation and carefully observe the output. Use the output of the graph for the calculation in the following parts. You may use graph papers in any part if required.

A.2 Keep the detector stationary at $r_{\rm D} = \theta = 0$ in the simulation. Observe the output graph of $f(t)$ between the graph start and end time 5 s and 55 s respectively. Tabulate all the minima ($f_{\rm min}$ and the corresponding time t) in the range. Plot $f_{\rm min}$ vs t. 1.2pt

A.3 Obtain an expression for the minimum frequency that the detector will eventually detect if kept stationary. Express your answer in terms of v_s, ω, R, and the relevant variables. 1.0pt

You will be using various input values in the panel to obtain the answers for the following parts. In the table given on page A2-4, report the various values of $r_{\rm D}, \theta, v_{\rm D}, \gamma, t_i, t_f$, and Δ which you have used to obtain the quantity or any intermediate equation. Also, mention the corresponding quantity or that equation against the input values.

Any formulae/equations in solving this part must be mentioned in the working sheets in detail. Marks will not be awarded solely based on the final answer. If you want to use a graph, mention the graph paper number on the answer sheet. Note that there is no error estimation required in this question.

Solve the next four questions in any order that you find appropriate.

A.4 Obtain the source's initial coordinates (X_A, Y_A) i.e. coordinates of the point A at time $t = 0$ in meters. 1.4pt

A.5 Obtain the values of f_0, ω, R, v_s. 2.1pt

A.6 Obtain the value of β in degrees. 2.0pt

A.7 Obtain the source's initial center coordinates (X_C, Y_C) i.e. coordinates of the point C in meters. 2.1pt