Snell's Law

Part A: Light Propagation Through a Semi-Sphere

A.1 [0.5pt] Using Snell's law, larger angle of refraction corresponds to larger refractive index that the light sees in the semi-sphere (n_1). [0.2pt]

This means that the colour with a smaller angle of refraction propagates faster in the semi-sphere, which is colour *a*. [0.3pt]

A.2 [1.0pt] The light rays no longer exit the bottom of semi-sphere when the ray is refracted by 90° with respect to the normal. [0.2pt]

Using Snell's law,

$$n_x sin\theta_i = n_2 \sin(90°) \quad [0.2pt]$$

Replace "x" with "a" and "b" for the condition of ray a and ray b, respectively.

For ray a, $n_a sin(50°) = 1.0 \sin(90°)$

$n_a = 1.305$ [0.2pt]

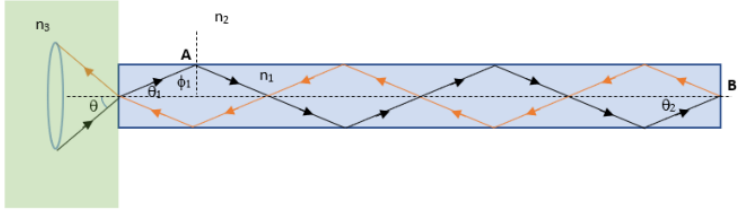
For ray b, $n_b sin(45°) = 1.0 \sin(90°)$

$n_b = 1.414$ [0.2pt]

Difference in refractive index, Δn = **0.109** [0.2pt]

Part B: Light Propagation Through a Cylindrical Rod

B.1 [2.0pt] Given $n_1 = 1.5$, $n_2 = 1.0$, $n_3 = 1.4$,



Identify 2 conditions that need to be obeyed: [0.2pt]

#1: Light incident at point A should be totally reflected.

$$n_1 sin\phi_1 > n_2 \sin(90°)$$

$$\phi_1 > sin^{-1}\frac{1.0}{1.5} = 41.81° \quad [0.3pt]$$

The condition for angle $\theta_1 < 90° - 41.81° = 48.19°$ [0.2pt]

Using Snell's law,

$$\theta < sin^{-1}\frac{1.5}{1.4}\sin(48.19°) = 53° \quad [0.2pt]$$

B.1 [cont.] **#2: Light incident at point B should be totally reflected.**

$$n_1 sin\theta_2 > n_2 \sin(90°)$$

$$\theta_2 > sin^{-1}\frac{1.0}{1.5} = 41.81° \quad [0.3pt]$$

The condition for angle $q_1 > 41.81°$ [0.2pt]

Using Snell's Law

$$\theta > sin^{-1}\frac{1.5}{1.4}\sin(41.81°) = 45.58° \quad [0.2pt]$$

Therefore, the incident angle, q, where light is totally reflected back to the polymer is $45.58^o < q < 53^o$. [0.4pt]

B.2 (i) [0.6pt] When right end of rod is coated with oil with $n_4 = 1.6$, condition #2 changes. [0.2pt]

Light incident on point B will always be refracted. [0.2pt]

Light CANNOT be totally reflected back to the polymer. [0.2pt]

B.2 (ii) [0.9pt] When the setup is placed in water, $\mathbf{n_2 = 1.33}$. Both condition #1 and #2 change. [0.2pt]

#1:

$$\phi_1 > sin^{-1}\frac{1.33}{1.5} = 62.46^\circ \quad \text{[0.2pt]}$$

$\theta_1 < 90^o - 62.46^o = 27.54^o$

Using Snell's law,

$$\theta < sin^{-1}\frac{1.5}{1.4}\sin(27.54^\circ) = 29.7^\circ$$

#2:

$$\theta_2 > sin^{-1}\frac{1.33}{1.5} = 62.46^\circ$$

Using Snell's law,

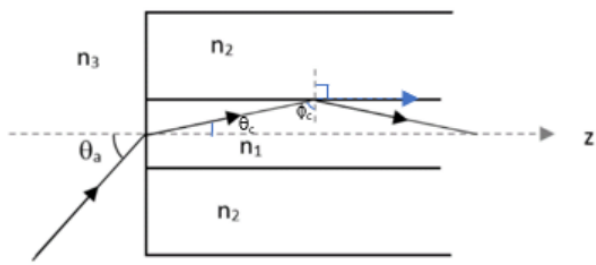
$$\theta > sin^{-1}\frac{1.5}{1.4}\sin(62.46^\circ) = 71.81^\circ \quad \text{[0.2pt]}$$

As conditions #1 and #2 does not overlap. [0.1pt]

Again, light CANNOT be totally reflected back to the polymer. [0.2pt]

Part C: Light Propagation Through an Optical Fibre

C.1 [2.0pt]



In order for light to not refractive out from the fiber, the angle of refraction that the light makes with the normal of the fiber core/cladding boundary needs to be 90^o. Using Snell's law,

$$n_1 sin\phi_c = n_2 \sin(90^\circ) \quad .....(1) \quad \text{[0.4pt]}$$

Taking the square of eq. (1), we have,

$$n_1^2 sin^2\phi_c = n_2^2$$

Using the trigonometry identity, we have, [0.2pt]

$$n_1^2(1 - cos^2\phi_c) = n_2^2$$

Rearranging, we got,

C.1 [cont.]

$$n_1^2 cos^2\phi_c = n_1^2 - n_2^2$$

Or, [0.2pt]

$$n_1^2 sin^2\theta_c = n_1^2 - n_2^2 \quad .....(2)$$

Taking the square root of eq. (2), we have,

$$n_1 sin\theta_c = \sqrt{n_1^2 - n_2^2} \quad [0.4pt]$$

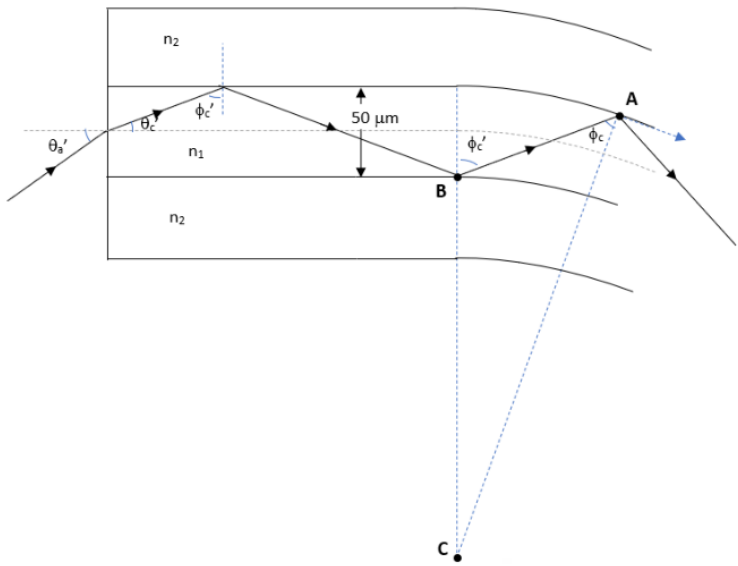
With this, we can use Snell's law to calculate the maximum angle of light incident,

$$n_3 sin\theta_a = n_1 sin\theta_c = \sqrt{n_1^2 - n_2^2} \quad [0.4pt]$$

$$\theta_a = sin^{-1} \frac{\sqrt{n_1^2 - n_2^2}}{n_3} \quad [0.4pt]$$

or $\theta_a = \sin^{-1} \sqrt{{n_1}^2 - {n_2}^2}$

C.2 [2.6pt]



- Identifying the correct relationship between $\phi_c$and ϕ_c' [0.4pt]

- Find the relationship between ϕ_c and ϕ_c':

- Using the Law of Sine,

$$sin\phi_c = \frac{CB}{CA} sin(\pi - \phi_c{}') \quad [0.4pt]$$

- For light guiding at point A,

$\sin \phi_c = \frac{n_2}{n_1}$ [0.4pt]

CB = 10000−25 = 9975 μm

CA = 10000+25 = 10025 μm [0.4pt]

C.2 [cont.]

$$sin(\pi - \phi_c{}') = \sin(\phi_c')$$

Therefore,

$$\sin(\phi_c') = \frac{10025 n_2}{9975 n_1} \quad [0.4pt]$$

Rearrange into eq.(1) and derive following part (a):

$$a.n_1 \sin(\phi'_c) = n_2, \qquad where\ a = \frac{9975}{10025}$$

$$a^2\, n_1^2 \sin^2(\phi'_c) = n_2^2$$

$$a^2 n_1^2 (1 - cos^2 \phi'_c) = n_2^2 \qquad \text{[0.4pt]}$$

$$a^2 n_1^2 cos^2 \phi'_c = a^2 n_1^2 - n_2^2$$

$$a^2 n_1^2 sin^2 \theta'_c = a^2 n_1^2 - n_2^2$$

$$n_1 sin\theta'_c = \frac{1}{a}\sqrt{a^2 n_1^2 - n_2^2}$$

To calculate the new angle of acceptance,

$$n_3 sin\theta'_a = n_1 sin\theta'_c = \tfrac{1}{a}\sqrt{a^2 n_1^2 - n_2^2}$$

$$\theta'_a = sin^{-1}\left(\tfrac{10025}{9975 n_3}\sqrt{a^2 n_1^2 - n_2^2}\right) \qquad \text{[0.2pt]}$$

C.3 [0.4pt] For case C.1: State that $n_3 = 1$;

$$\sqrt{n_1^2 - n_2^2} = \sqrt{1.45^2 - 1.44^2} = 0.17 \qquad \text{[0.1pt]}$$

$$\theta_a = sin^{-1} 0.17 = 9.79^0 \qquad \text{[0.1pt]}$$

For case C.2: Substitute,

$$\theta'_a = sin^{-1}\left(\frac{10025}{9975}\sqrt{\left(\tfrac{9975}{10025}\right)^2 1.45^2 - 1.44^2}\right) \qquad \text{[0.1pt]}$$

$$= 5.15^\circ \qquad \text{[0.1pt]}$$