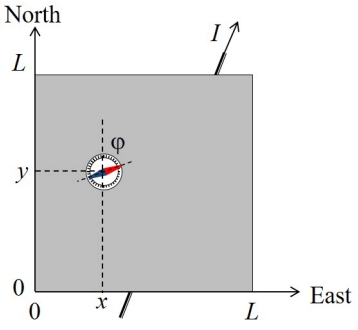
E1: Hidden wire

Experimental setup and tasks

A very long copper wire runs horizontally at unknown depth h under a horizontal square surface of side length $L = 100.0\,\text{mm}$. The sides of the square are oriented West-East (the x-axis) and South-North (the y-axis), as shown in the figure. The origin of the coordinate system coincides with the South-West corner of the square.



The wire is connected to an adjustable DC source (not shown in the figure), which can provide a current I in the range from $-5\,\text{A}$ to $5\,\text{A}$. The reversal of the sign of the current corresponds to a reversal of the polarity of the source. A small compass can be placed on the square surface, (including its circumference) to sense the magnetic field of the wire through the deflection angle φ between the magnetic needle and the North (y) direction. Positive φ values correspond to an Eastward deflection, as shown in the figure, while negative φ correspond to a Westward deflection. You can assume that:

- The magnetic needle is a point-like magnetic dipole, which can rotate freely around the vertical axis, i.e. the compass is sensitive to the horizontal component of the magnetic field only.
- The height of the needle above the surface is negligible compared to the depth of the wire beneath the surface, i.e. the needle is situated in the xy-plane.

Design your experiment and make the necessary simulations to perform the following tasks:

a. Determine the orientation of the wire with respect to the coordinate system by specifying its equation in the form: $y = ax + b$, and estimate the uncertainties of the parameters a and b. Draw the wire position on a graph and indicate the direction corresponding to a positive current I.

b. Determine the depth h of the wire below the surface and the horizontal component B_E of the Earth's magnetic field. In this task you are not required to calculate the experimental uncertainties explicitly, however, your final results must be represented with an appropriate number of significant digits.

The magnetic permeability of free space is

$$\mu_0 = 4\pi \times 10^{-7}\,\text{T m/A}.$$

Description of the simulation software

The command line program simulates the measurement of the deflection angle φ after providing the current I and placing the compass at the coordinates x and y on the surface.

A typical output of a single simulation cycle of the program looks like:

```
Enter I (A) between -5.0 and 5.0: 3.4
Enter X (mm) between 0 and 100: 55
Enter Y (mm) between 0 and 100: 31
PHI = -33 degrees
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Enter I (A) between -5.0 and 5.0: _
```

First, you enter the current I in A (the number between -5.0 and 5.0), then coordinates x and y in mm (the numbers between 0 and 100). Each input is confirmed with the **Enter** key. The program will output the value of φ (`PHI`) in degrees (rounded to $1°$) and return to the initial prompt.

The current input I will be rounded to 0.1 A, the coordinate inputs x, y will be rounded to 1 mm before being used in simulation. (There is no point in trying to input more precise numbers).

Every time you change the position of a compass, its real position used in simulation differs from the input coordinates with an error about 0.5 mm. (It is a simulation of a limited precision when you place an object).

Any time you need to quit the program, press **Ctrl**+**C**.