# E2: Hot Cylinder

Start the experiment with the heater on full for 300 Watts and the thermostats located evenly across the length of the rod and display the results every 100 seconds. Then plan out the remainder of the experiment while waiting, or do the other experiment. The rod reaches steady state at about 600 seconds. Find the average temperature at the five thermostats by considering the last five measurements; you will use this later.

The most accessible approach is then to study the steady state behavior, the uniform temperature behavior, the low temperature behavior, and the high temperature behavior. Separating the low and high temperature behaviors is useful because blackbody radiation dominates at higher temperatures while convective loss is most significant at near room temperature.

Finding the heat capacity is done by heating the rod at a low enough rate for a short enough time so that heat loss is as small as possible.

One possibility is to give a total of 1500 J of heat, but at various power settings and various times, while keeping the temperature as low as possible.

The average temperature of the rod is computed from the five equally spaced points by applying Simpson's rule,

$$T_{avg} = \frac{T_1 + 4T_2 + 2T_3 + 4T_4 + T_5}{12}$$

Computing instead a direct average yields a $+5\%$ error.

It is found that the average temperature for heating times less than 50 seconds is $55.4 \pm 0.5$ °C, yielding specific heat capacity of $c = 114 \pm 1$ J/kg K.

Heat the rod full power for 600 seconds, and then allow to cool.

The rod temperature becomes uniform at about 700 seconds. Average the five points to obtain an average rod temperature.

Linear cooling predicts a straight line graph for $\ln(T - T_0)$ versus $t$

The convective heat loss rate is then given by $A\alpha(T - T_0)$, where $A$ is the surface area of the rod. Do not forget the end caps!

The radiative heat loss rate is $\beta\sigma(T^4 - T_0^4)$ where $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/(m^2\,K^4)}$. The radiative heat loss rate is then given by $A\beta\sigma(T^4 - T_0^4)$.

Note that at temperatures close to $T_0$ the radiative expression can be written as

$$A\beta\sigma(T^4 - T_0^4) \approx A\beta\sigma 4(T - T_0)T_0^3$$

This means that the linear heat loss rate at temperatures close to $T_0$ is

$$A\left(\alpha + \beta\sigma 4T_0^3\right)(T - T_0)$$

For the uniform, low temperature cooling rod,

$$mc\frac{\mathrm{d}T}{\mathrm{d}t} = -A\left(\alpha + \beta\sigma 4T_0^3\right)(T - T_0)$$

The solution is of the form

$$T - T_0 = Ce^{-Bt}$$

where

$$B = A\frac{\alpha + \beta\sigma 4T_0^3}{mc}$$

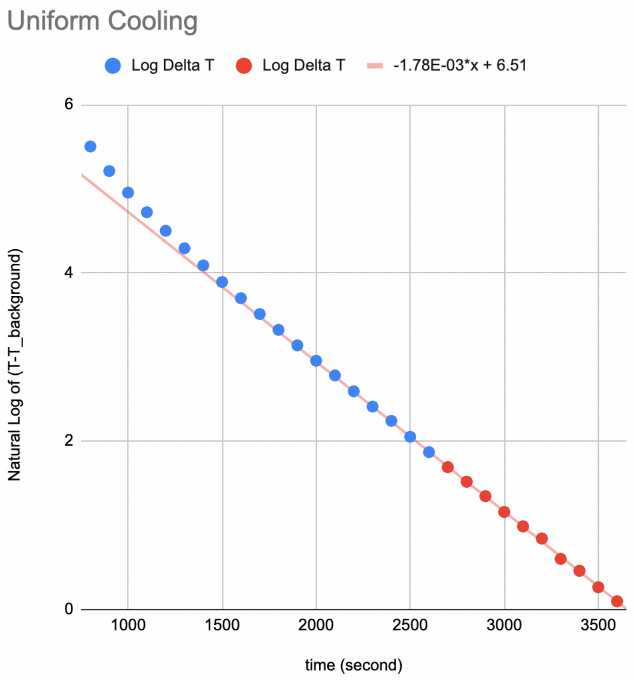
On a log plot of $\ln(T - T_0)$ as a function of time $t$, the plot should be linear, with a slope given by

$$-A\frac{\alpha + \beta\sigma 4T_0^3}{mc}$$

It is also possible to plot $\mathrm{d}T/\mathrm{d}t$ as a function of $T - T_0$, and the plot will be linear, with a slope also given by

$$-A\frac{\alpha + \beta\sigma 4T_0^3}{mc}$$

The slope in either case is found to be $-1.78 \times 10^{-3}$ /s.




Note that only the last points (in red) were used to determine the linear cooling line. It is clearly a good fit from $t = 2000$ s on, which corresponds to rod temperatures of $T < 45$ C.

To find the blackbody behavior we want to heat the rod as much as possible such that the blackbody heating becomes the dominant form of heat loss. Since the hot rod is in steady state, the heat radiated must be equal to 300 W. Use the results from the beginning.

The average temperature of the rod is computed from the five equally spaced points by

$$T_{avg} = \frac{T_1 + 4T_2 + 2T_3 + 4T_4 + T_5}{12} = 662\,°\mathrm{C}.$$

Computing a direct average yields a $+1.5\%$ error.

The average of $T^4$ is found from

$$T^4_{\text{avg}} = \frac{{T_1}^4 + 4{T_2}^4 + 2{T_3}^4 + 4{T_4}^4 + {T_5}^4}{12} = 7.95 \times 10^{11}\ \mathrm{K}^4$$

Computing a direct average yields a $+6.3\%$ error.

The rate of linear temperature heat loss is found from above to be

$$(-1.78 \times 10^{-3}\ /\mathrm{s})mc\Delta T = 59\ \mathrm{W}$$

The blackbody remainder term is then

$$300 - 59 = 241 \text{ W},$$

and necessarily equals

$$A\beta\sigma(T^4 - T_0^4) - A\beta\sigma 4T_0^3(T - T_0),$$

where the second term reflects the fact that we had considered part of the blackbody behavior as being linear.

Solving, $\beta = 0.304 \pm 0.004$.

Failing to subtract the second term would yield $\beta = 0.28$.

We are now in a position to find $\alpha$, from

$$-A\frac{\alpha + \beta\sigma 4T_0^3}{mc} = -1.78 \times 10^{-3} \text{ /s}$$

which yields $\alpha = 2.93$

Alternatively, for the uniform, high temperature cooling rod,

$$mc\frac{\mathrm{d}T}{\mathrm{d}t} \approx -A\beta\sigma\left(T^4 - T_0^4\right)$$

as the radiative cooling effect will dominate.

On a plot of $\mathrm{d}T/\mathrm{d}t$ as a function of $T^4 - T_0^4$, the plot should be linear, with a slope given by

$$-\frac{A\beta\sigma}{mc}$$

The slope is found to be $-7.8 \times 10^{-12}$ K$^3$/s

This means $\beta/c = 3.25 \times 10^{-3}$ kg K/J; this gives $\beta = 0.36$, which is too high; ignoring the linear loss effects was significant; as was previously seen, almost 20% of the heat loss is from convection in this temperature range.

We can use the high temperature behavior to find the heat flux through the center of the rod. The average of $T$ and $T^4$ on the non-heated half of the rod is $599$ C and $5.8 \times 10^4$ K$^4$, yielding a heat loss at $112$ W. That heat necessarily came from the other side of the rod.

The temperature gradient is $-898$ K/m, so $k = 397$ W/mK. Don't forget that the formula provided gave the rate of heat flux, which means that we needed to consider the cross sectional area of the wire.

## Marking scheme

**Finding $c$, 2.5 pt total**

| | Task | Pts |
|---|---|---|
| 2.1 | Idea of heating the rod by a fixed $Q$. | 0.6 |
| 2.2 | Obtaining an equation relating the inserted heat with the temperature change and $c$. | 0.4 |
| 2.2 | Heating the rod for a short duration for mitigating the effects of heat loss: | |
| | heating for less than 60 seconds. | 0.2 |
| 2.3 | Averaging the temperature of the rod: | |
| | averaging over one to three points; | 0.1 |
| | Use Simpson rule (or equivalent) | 0.3 |
| | averaging over four or more points; | 0.2 |
| 2.4 | Checking more than one time value | 0.2 |
| 2.5 | Numerical value of $c$: | |
| | $c$ within [103;123] J/(K kg); | 0.3 |
| | $c$ within [108;118] J/(K kg). | 0.2 |

**Finding the linear heat loss, 2.0 pt total**

| | Task | Pts |
|---|---|---|
| 3.1 | Idea of looking at how the rod cools down at the low temperature limit (with no heating). | 0.2 |
| 3.2 | Obtaining an equation for $T$ as a function of $t$ in terms of $\alpha$, $\beta$, and $c$: | |
| | linearize radiative loss around $T_0$; | 0.3 |
| | obtaining a differential equation for $T(t)$; | 0.1 |
| | solving the differential equation to get $T(t)$. | 0.2 |
| 3.3 | Finding the average temperature at $t$: | |
| | averaging over one to three points; | 0.1 |
| | Use Simpson rule (or equivalent) | 0.3 |
| | averaging over four or more points; | 0.1 |
| 3.4 | Graphically finding the slope (which is a function of $\alpha$, $\beta$, and $c$): | |
| | Plot 2 to 4 points in range $T < 50$C; | 0.1 |
| | Plotting 5 or more points in range $T < 50$C; | 0.1 |
| | axes labels and units; | 0.1 |
| | axes tickmarks with values. | 0.1 |
| 3.5 | Numerical value of the slope: | |
| | slope within [$-1.58 \times 10^{-3}$,$-1.98 \times 10^{-3}$]; | 0.2 |
| | slope within [$-1.68 \times 10^{-3}$,$-1.88 \times 10^{-3}$]; | 0.1 |

**Finding $\beta$, 2.5 pt total**

| | Task | Pts |
|---|---|---|
| 4.1 | Idea of looking at the steady state at the high temperature limit. | 0.2 |
| 4.2 | Writing down the heat balance: | |
| | accounting for the area of the end caps; | 0.1 |
| | accounting for the linear contribution to the heat loss by removing the $\alpha$ dependence from the previously found slope; | 0.2 |
| | final expression for $\beta$ in terms of mean value of $T$ and $T^4$ of the steady state. Making a mistake in the previous parts shouldn't be penalised here. | 0.2 |
| 4.3 | Measurements: | |
| | Heating power sufficiently big such that the steady state temperature is bigger than 500 °C; | 0.1 |
| | Waiting for at least 300s to reach the steady state | 0.1 |
| | Waiting for at least 600s to reach the steady state | 0.1 |
| 4.4 | Finding the average temperature: | |
| | averaging over one to three points; | 0.1 |
| | Use Simpson rule (or equivalent) | 0.3 |
| | averaging over four or more points; | 0.1 |
| 4.5 | Finding the average $T^4$ (for calculating average radiative loss): | |
| | averaging over one to three points; | 0.1 |
| | Use Simpson rule (or equivalent) | 0.3 |
| | averaging over four or more points; | 0.1 |
| 4.6 | Numerical value of $\beta$: | |
| | $\beta$ within [0.25;0.35]; | 0.3 |
| | $\beta$ within [0.28;0.32]. | 0.2 |

**Finding $\alpha$, 0.5 pt total**

| | Task | Pts |
|---|---|---|
| 5.1 | Obtaining an expression for $\alpha$ in terms of the slope $\gamma$. | 0.1 |
| 5.2 | Numerical value of $\alpha$: | |
| | $\alpha$ within [2.33;3.23]$\mathrm{W/(m^2\,K)}$; | 0.2 |
| | $\alpha$ within [2.53;3.03]$\mathrm{W/(m^2\,K)}$. | 0.2 |

**Finding $k$, 2.5 pt total**

| | Task | Pts |
|---|---|---|
| 6.1 | Idea of looking at the flux from part of the rod to the other | 0.4 |
| 6.2 | Theory: | |
| | Expressing heat flux in terms of $k$ and the temperature gradient; | 0.2 |
| | Expressing heat flux in terms of the average $T$, $T^4$, and the heating power of one of the halves of the rod; | 0.4 |
| | accounting for the area of the end caps. | 0.1 |
| 6.3 | Finding the average temperature of one of the halves: | |
| | averaging over one to three points; | 0.1 |
| | averaging over four or more points. No marks if points not equally spaced and average doesn't account for the unevenness; | 0.1 |
| 6.4 | Finding the average $T^4$ (for calculating average radiative loss): | |
| | averaging over one to three points; | 0.1 |
| | averaging over four or more points; | 0.1 |
| 6.5 | Finding the temperature gradient: | |
| | Using at least two points for the gradient calculation; | 0.1 |
| | Using $(f(x+h)-f(x-h))/2h$ for numerical derivative; | 0.2 |
| | Having the range of points used for gradient calculations not farther apart than 5 cm; | 0.1 |
| | Having the range of points used for gradient calculations not closer than 1 cm; | 0.1 |
| 6.5 | Numerical value of $k$: | |
| | $k$ within [328;488]$\mathrm{W/(m\,K)}$; | 0.3 |
| | $k$ within [378;438]$\mathrm{W/(m\,K)}$. | 0.2 |

**Some grading notes:**

- Failure to record and report the location of the sensors will result in a penalty of **-1.0 pt** *for each occurrence!*. It is acceptable to clearly state the location of the sensors in one part of the report, and then mentioning that they are not moved during the experiment.
- When computing spatial averages, if the spacing between thermometers is not uniform, the averaging techniques must use appropriate weighting, or there is a penalty of **-0.1 pt** *for each occurrence!*
- When computing spatial averages, if the rod is not mostly uniform in temperature, a Simpson's Rule technique or equivalent must be used to obtain the 0.3 pts. If instead all of the temperatures are within two error limits, then Simpson is not required to obtain the 0.3 pt.
- Any numerical derivatives must use the symmetric form

$$f'(x) \approx (f(x+h)-f(x-h))/2h$$

or some equivalent, or better, method.