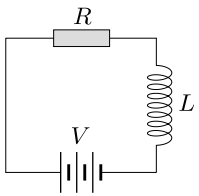
T2: Thermal oscillations (10 pts)

A resistor is made of a material which undergoes a phase transition so that its resistance takes one of the two values, R_1 if its temperature is smaller than T_c, and $R_2 > R_1$ if the temperature is larger than T_c.



This resistor is connected to a voltage source through an inductor of inductance L. It appears that if the applied voltage V is between two critical values, $V_1 < V < V_2$, the temperature of the resistor starts oscillating. Assume that (i) the heat flux P from the resistor to the ambient medium is given by $P = \alpha(T - T_0)$, where α is a constant, T denotes the temperature of the resistor, and T_0 is the ambient temperature; (ii) the geometrical size of the resistor is so small that it will reach a thermal equilibrium much faster than the characteristic time L/R_2.

(a) *(2 pts)* Express V_1 and V_2 in terms of the other parameters defined above.

(b) *(6 pts)* Assuming that $V_1 < V < V_2$, sketch qualitatively how the temperature of the resistor T depends on time t, and find the ratio $(T_{\text{max}} - T_0)/(T_{\text{min}} - T_0)$, where T_{max} and T_{min} denote the maximal and minimal values of T, respectively.

(c) *(2 pts)* Find the period of oscillations if $V = \sqrt{V_1 V_2}$ and $R_2 = 16R_1$.