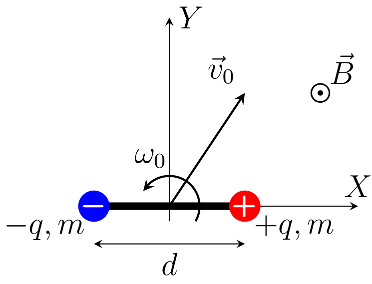
T3: Dipole in a magnetic field (10 pts)

Two small balls of mass m each with charges $+q$ and $-q$ respectively, connected by a rigid massless rod of length d, form a dipole. The dipole is parallel to plane XY and is placed in a uniform magnetic field $\vec{B}$ perpendicular to XY.



Initially, the dipole is aligned with the direction X and has initial angular velocity ω_0 in plane XY, as shown. Its center of mass is initially located at origin and given initial velocity $\vec{v}_0$ parallel to XY, as well.

Consider three distinct scenarios (a, b, c-d):

(a) *(2 pts)* Find ω_0 and the direction of $\vec{v}_0$, so that the center of mass will move with the constant velocity $\vec{v} = \vec{v}_0$?

(b) *(3 pts)* Given ω_0, find such $\vec{v}_0$ (direction and magnitude), so that the center of mass will travel in a circle. Find the circle radius R_c and the coordinates x_c, y_c of its center. You don't need to prove the uniqueness of the solution.

(c) *(4 pts)* Given $\vec{v}_0 = 0$, find the minimal $\omega_0 = \omega_{\text{min}}$ necessary for the dipole to reverse its orientation during the motion.

(d) *(1 pt)* If the dipole starts with $\vec{v}_0 = 0$ and $\omega_0 = \omega_{\text{min}}$ found in part (c), the trajectory of its center of mass has an asymptote. Find the distance D from the origin to the asymptote.

Useful vector identity:

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}\,(\vec{a} \cdot \vec{c}) - \vec{c}\,(\vec{a} \cdot \vec{b}),$$

where "$\times$" and "$\cdot$" denote vector product and scalar product respectively.