

T2: Spaceships (10 pts)

Alice and Bob are twin astronauts on a long space mission. After many years, they are finally approaching each other to reunite. Alice's spaceship is moving towards Bob's spaceship at a speed of $u = \frac{3}{5}c$, where c is the speed of light.

During their approach, both Alice and Bob send gifts to each other. Alice sends gifts to Bob at regular time intervals Δt_0 in her own frame of reference, with each gift travelling at a velocity $v = \frac{4}{5}c$ (again, in her frame of reference). Similarly, Bob sends gifts to Alice at the same regular time intervals Δt_0 in his own frame of reference, with each gift also travelling at a velocity $v = \frac{4}{5}c$ in his frame of reference. Assume that the distance L between Alice and Bob is so large that there are many gifts in transit at any given moment.

- a) (5 pts) In Bob's reference frame, find
- (i) the distance between two successive gifts sent by Alice, and
 - (ii) the time interval Δt_1 at which these gifts from Alice arrive at Bob's spaceship.
- b) (5 pts) At a given instant, Alice can see a number of gifts moving away from her and a number of gifts moving towards her. What is the ratio between these two numbers?