

T2: Spaceships (10 pts)

Alice and Bob are twin astronauts on a long space mission. After many years, they are finally approaching each other to reunite. Alice's spaceship is moving towards Bob's spaceship at a speed of $u = \frac{3}{5}c$, where c is the speed of light.

During their approach, both Alice and Bob send gifts to each other. Alice sends gifts to Bob at regular time intervals Δt_0 in her own frame of reference, with each gift travelling at a velocity $v = \frac{4}{5}c$ (again, in her frame of reference). Similarly, Bob sends gifts to Alice at the same regular time intervals Δt_0 in his own frame of reference, with each gift also travelling at a velocity $v = \frac{4}{5}c$ in his frame of reference. Assume that the distance L between Alice and Bob is so large that there are many gifts in transit at any given moment.

a) (5 pts) In Bob's reference frame, find

(i) the distance between two successive gifts sent by Alice, and

(ii) the time interval Δt_1 at which these gifts from Alice arrive at Bob's spaceship.

b) (5 pts) At a given instant, Alice can see a number of gifts moving away from her and a number of gifts moving towards her. What is the ratio between these two numbers?