T1: Sunny (10 pts)

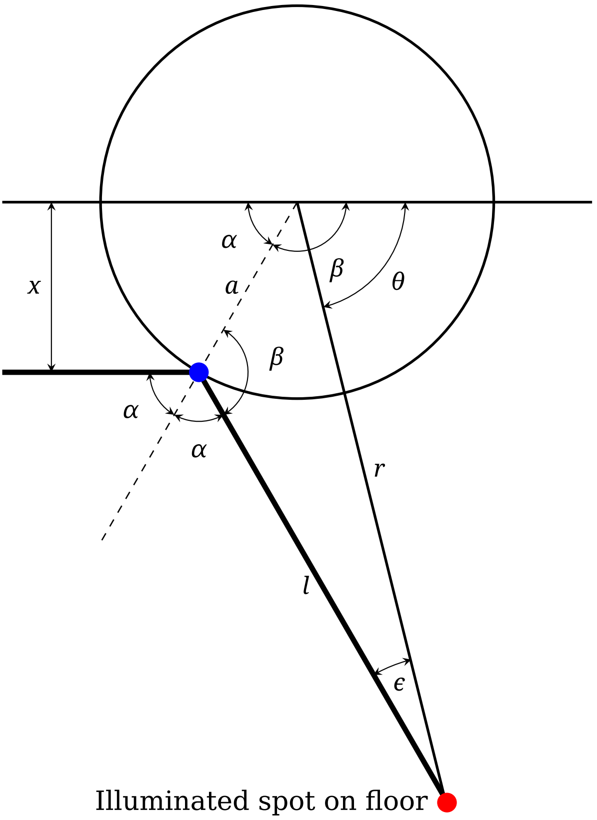

Figure 1: Top view of a light ray hitting the chair leg. The incoming light ray strikes the cylinder at the blue dot, and the reflected ray strikes the floor at the red dot.

Part a) Since points A and B are very close, I_0 is the illuminance surplus due to direct sunlight.

Consider a small (infinitesimal) horizontal rectangle $[x, x+\delta x] \times [y, y+\delta y]$ placed in the light beam. As depicted in Fig 1, we let l be the horizontal distance between the point where a ray hits the cylinder and where it hits the floor and α the horizontal angle between the ray and the surface normal. The spot on the floor made by the light makes an angle of 2α from the incident ray, and forms a small patch of size $(l2\delta\alpha)(\delta l)$ on the floor.

Considering the side view sketched in Figure 2, we see that $\delta l \propto \delta y$ for both the ray that hits the ground directly, or a ray that is reflected before hitting the ground.

We thus have

$$I_0 \delta x = 2Il\delta\alpha \tag{1}$$

Let $\beta = \pi - \alpha$ as sketched in Fig 1. Then

$$\delta x = a\left(\sin(\beta + \delta\beta) - \sin\beta\right) \approx a\cos\beta\delta\beta \tag{2}$$

Therefore,

$$I = -\frac{I_0 a}{2l}\cos(\beta) \tag{3}$$

(Note that $\cos\beta < 0$.) This expression is accurate to order $(a/l)^2$, but is not in terms of the variables re-

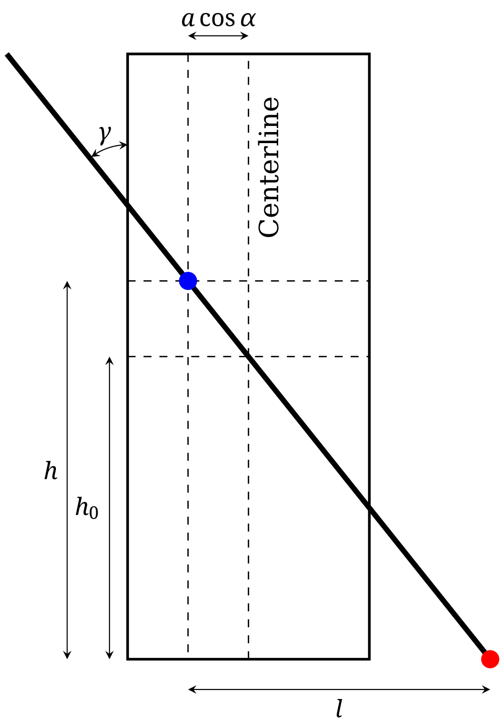

Figure 2: Side view, with the reflected light ray rotated to lie in the same plane as the incident ray.

quested. A more accurate expression would be

$$I = -\frac{I_0 a}{\sqrt{4l^2 + a^2\sin^2(\beta)}}\cos(\beta),$$

but it is not necessary to find this expression.

To obtain I as a function of r and θ, we need to express β and l in terms of r and θ.

If $a \ll r$, l and r almost coincide. To leading order thus $l \approx r$ and $\theta \approx \pi - 2\alpha = 2\beta - \pi$. Substituting l and β in (3) results in

$$I \approx \frac{I_0 a}{2r}\sin(\theta/2) \tag{4}$$

Part b) The rings appear because the fingers block the light. For the middle ring, the finger is approximately horizontal.

For l_0 be the horizontal distance l for $\alpha = \pi/2$. Consider the sketch in Fig. 2. Note that

$$l = l_0 + a\cos\alpha = l_0 + a|\cos(\beta)| \tag{5}$$

The minimum value $R_{\min}$ is attained at $\alpha = \beta = \pi/2$, where $\theta \approx 0$ and $R_{\min} \approx l_0$ within an error of order $(a/r)^2$. Hence,

$$l \approx R_{\min} + a\cos\alpha = R_{\min} + a|\cos(\beta)|$$

The Cosine theorem applied to the triangle between the center of the chair leg, the point where the ray reflects form the leg and the point where it hits the floor, gives the relation

$$r^2 = a^2 + l^2 - 2al\cos(\beta) \tag{6}$$

Using (6) and expanding, we obtain

$$r^2 = R_{\min}^2 + 4aR_{\min}\cos\alpha + 3a^2\cos^2\alpha$$

Using $\cos\alpha \approx \sin(\theta/2)$, and dropping terms of order $(a/r)^2$, we arrive at

$$R - R_{\min} \approx 2a\sin(\theta/2) \tag{7}$$

The factor of 2 is significant!

Marking scheme T1

Problem T1 a)			**Pts**
A	Correct angles, $2\alpha+\theta>\pi$		0.5
	Only state $2\alpha+\theta=\pi$ without justification		0.2
B	Reflection law (stated in any way)		0.5
C	$x = a\sin\alpha$		0.5
D	Eq. 1		1.0
	Missing factor of 2	-0.3	
	r instead of l	-0.3	
E	Eq. 3 for I		0.5
F	Justify $l \approx r$		0.5
	Assume $l \approx r$ without justification		0.2
G	Eq. 4 for I		1.5
Total on T1 a)			**5.0**

Problem T1 b)		**Pts**
H	Cosine Law expression	1.0
I	Eq. 5 for l	1.0
	qualitative understanding why $R(\theta)$ varies with θ	0.5
J	Justifying $l_0 \approx R_{\min}$ to leading order	1
	Only stating $l_0 \approx R_{\min}$	0.5
K	Final Eq. 7	2.0
	Writing $R - R_{\min} = a\sin(\theta/2)$	1.0
	Only has $R_{\max} - R_{\min} = 2a$	0.5
	Only has $R_{\max} - R_{\min} = a$	0.0
Total on T1 b)		**5.0**

General rules for marking in T1:

- The grain size for marking is 0.1 Pts.
- Yellow shaded categories receive a single mark
- Partial marks can be awarded for most aspects.
- For each mistake in calculation (algebraic or numeric) 0.3 Pts. are deducted.
- If a mistake leads to a dimensionally incorrect expression no marks are given for the result.
- Propagating errors are not punished again unless they are dimensionally wrong or entail oversimplified/wrong physics (e.g. neglecting friction effects).
- Getting a correct answer for a later part that could only be arrived at by doing a previous part correctly will result in the minimum score(s) for previous part(s) that would support the end result.
- Justifying F can be done graphically, algebraically, with words, but needs more than just writing $r \approx l$.
- Arriving at a correct G without supporting work would score 4.1 for A-G as (0.2, 0.5, 0.5, 0.7, 0.5, 0.2, 1.5).
- Arriving at an incorrect G without supporting work would score 0 for A-G.

- Arriving at a correct K without supporting work would score 4.5 for L-O as (1.0, 1.0, 0.5, 2.0)
- Arriving at an incorrect K without supporting work would score 0 for H-J, and then the points for K.