

**Problem 6.** Let  $\mathbb{Q}$  be the set of rational numbers. A function  $f: \mathbb{Q} \rightarrow \mathbb{Q}$  is called *aquaesulian* if the following property holds: for every  $x, y \in \mathbb{Q}$ ,

$$f(x + f(y)) = f(x) + y \quad \text{or} \quad f(f(x) + y) = x + f(y).$$

Show that there exists an integer  $c$  such that for any aquaesulian function  $f$  there are at most  $c$  different rational numbers of the form  $f(r) + f(-r)$  for some rational number  $r$ , and find the smallest possible value of  $c$ .