Solution

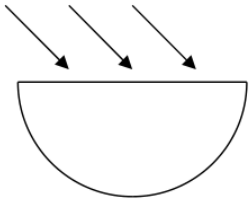


Figure 4

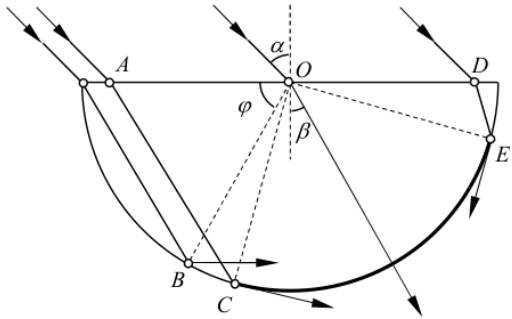


Figure 5

Let us use angle φ to describe the position of the rays in the glass (*Fig. 5*). According to the law of refraction $\sin 45° / \sin \beta = \sqrt{2}$, $\sin \beta = 0.5$, $\beta = 30°$. The refracted angle is 30° for all of the incoming rays. We have to investigate what happens if φ changes from 0° to 180°.

It is easy to see that φ can not be less than 60° ($AOB\angle = 60°$). The critical angle is given by $\sin \beta_{crit} = 1/n = \sqrt{2}/2$; hence $\beta_{crit} = 45°$. In the case of total internal reflection $ACO\angle = 45°$, hence $\varphi = 180° - 60° - 45° = 75°$. If φ is more than 75° the rays can emerge the cylinder. Increasing the angle we reach the critical angle again if $OED\angle = 45°$. Thus the rays are leaving the glass cylinder if:

$$75° < \varphi < 165° \ ,$$

CE, arc of the emerging rays, subtends a central angle of 90°.