Reference: O. F. Kabardin, V. A. Orlov, in "International Physics Olympiads for High School Students", eds. V. G. Razumovski, Moscow, Nauka, 1985. (In Russian).

Question 1.

The blocks slide relative to the prism with accelerations $\mathbf{a}_1$ and $\mathbf{a}_2$, which are parallel to its sides and have the same magnitude a (see Fig. 1.1). The blocks move relative to the earth with accelerations:

$$\mathbf{w}_1 = \mathbf{a}_1 + \mathbf{a}_0; \tag{1.1}$$

$$\mathbf{w}_2 = \mathbf{a}_2 + \mathbf{a}_0. \tag{1.2}$$

Now we project $\mathbf{w}_1$ and $\mathbf{w}_2$ along the x- and y-axes:

$$w_{1x} = a\cos\alpha_1 - a_0; \tag{1.3}$$

$$w_{1y} = a\sin\alpha_1; \tag{1.4}$$

$$w_{2x} = a\cos\alpha_2 - a_0; \tag{1.5}$$

$$w_{2y} = -a\sin\alpha_2. \tag{1.6}$$

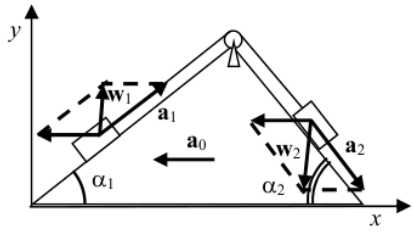


Fig. 1.1

The equations of motion for the blocks and for the prism have the following vector forms (see Fig. 1.2):

$$m_1\mathbf{w}_1 = m_1\mathbf{g} + \mathbf{R}_1 + \mathbf{T}_1; \tag{1.7}$$

$$m_2\mathbf{w}_2 = m_2\mathbf{g} + \mathbf{R}_2 + \mathbf{T}_2; \tag{1.8}$$

$$M\mathbf{a}_0 = M\mathbf{g} - \mathbf{R}_1 - \mathbf{R}_2 + \mathbf{R} - \mathbf{T}_1 - \mathbf{T}_2. \tag{1.9}$$

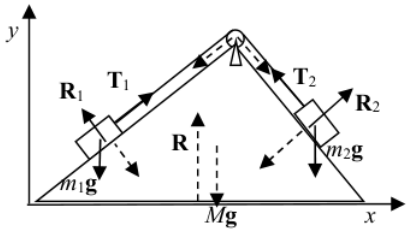


Fig. 1.2

The forces of tension $\mathbf{T}_1$ and $\mathbf{T}_2$ at the ends of the thread are of the same magnitude T since the masses of the thread and that of the pulley are negligible. Note that in equation (1.9) we account for the net force $-(\mathbf{T}_1+\mathbf{T}_2)$, which the bended thread exerts on the prism through the pulley**.** The equations of motion result in a system of six scalar equations when projected along x and y:

$$m_1 a\cos\alpha_1 - m_1 a_0 = T\cos\alpha_1 - R_1\sin\alpha_1; \tag{1.10}$$

$$m_1 a\sin\alpha_1 = T\sin\alpha_1 + R_1\cos\alpha_1 - m_1 g; \tag{1.11}$$

$$m_2 a\cos\alpha_2 - m_2 a_0 = -T\cos\alpha_2 + R_2\sin\alpha_2; \tag{1.12}$$

$$m_2 a\sin\alpha_2 = T\sin\alpha_2 + R_2\sin\alpha_2 - m_2 g; \tag{1.13}$$

$$-Ma_0 = R_1 \sin\alpha_1 - R_2 \sin\alpha_2 - T\cos\alpha_1 + T\cos\alpha_2; \tag{1.14}$$

$$0 = R - R_1\cos\alpha_1 - R_2\cos\alpha_2 - Mg. \tag{1.15}$$

By adding up equations (1.10), (1.12), and (1.14) all forces internal to the system cancel each other. In this way we obtain the required relation between accelerations a and a_0:

$$a = a_0 \frac{M + m_1 + m_2}{m_1\cos\alpha_1 + m_2\cos\alpha_2}. \tag{1.16}$$

The straightforward elimination of the unknown forces gives the final answer for a_0:

$$a_0 = \frac{(m_1\sin\alpha_1 - m_2\sin\alpha_2)(m_1\cos\alpha_1 + m_2\cos\alpha_2)}{(m_1 + m_2 + M)(m_1 + m_2) - (m_1\cos\alpha_1 + m_2\cos\alpha_2)^2}. \tag{1.17}$$

It follows from equation (1.17) that the prism will be in equilibrium ($a_0 = 0$) if:

$$\frac{m_1}{m_2} = \frac{\sin\alpha_2}{\sin\alpha_1}. \tag{1.18}$$