Solution Problem 1

The inertia moments of the three cylinders are:

$$I_1 = \frac{1}{2}\rho_1\pi\left(R^4 - r^4\right)h, \quad I_2 = \frac{1}{2}\rho_2\pi R^4 h = \frac{1}{2}mR^2 \quad , \; I3 = \frac{1}{2}\rho_2\pi\left(R^4 - r^4\right)h, \qquad (1)$$

Because the three cylinders have the same mass :

$$m = \rho_1\pi\left(R^2 - r^2\right)h = \rho_2\pi R^2 h \qquad (2)$$

it results:

$$r^2 = R^2\left(1 - \frac{\rho_2}{\rho_1}\right) = R^2\left(1 - \frac{1}{n}\right), n = \frac{\rho_1}{\rho_2} \qquad (3)$$

The inertia moments can be written:

$$I_1 = I_2\left(2 - \frac{1}{n}\right)\rangle I_2 \; , \quad I_3 = I_2\left(2 - \frac{1}{n}\right)\cdot\frac{1}{n} = \frac{I_1}{n} \qquad (4)$$

In the expression of the inertia momentum I_3 the sum of the two factors is constant:

$$\left(2 - \frac{1}{n}\right) + \frac{1}{n} = 2$$

independent of n, so that their products are maximum when these factors are equal: $2 - \frac{1}{n} = \frac{1}{n}$; it results n = 1, and the products $\left(2 - \frac{1}{n}\right)\cdot\frac{1}{n} = 1$. In fact n > 1, so that the products is les than 1. It results:

$$\mathrm{I_1} > \mathrm{I_2} > \mathrm{I_3} \qquad (5)$$

For a cylinder rolling over freely on the inclined plane (fig. 1.1) we can write the equations:

$$mg\sin\alpha - F_f = ma \qquad (6)$$

$$N - mg\cos\alpha = 0$$

$$F_f R = I\varepsilon \qquad (7)$$

where ε is the angular acceleration. If the cylinder doesn't slide we have the condition:

$$a = \varepsilon R \qquad (8)$$

Solving the equation system (6-8) we find:

$$a = \frac{g\sin\alpha}{1 + \dfrac{I}{mR^2}} \; , \quad F_f = \frac{mg\sin\alpha}{1 + \dfrac{mR^2}{I}} \qquad (9)$$

The condition of non-sliding is:

$$\mathrm{F_f} < \mu\mathrm{N} = \mu\mathrm{mg}\sin\alpha$$

$$\mathrm{tg}\alpha \; < \; \mu\left(1 + \frac{mR^2}{I_1}\right) \qquad (10)$$

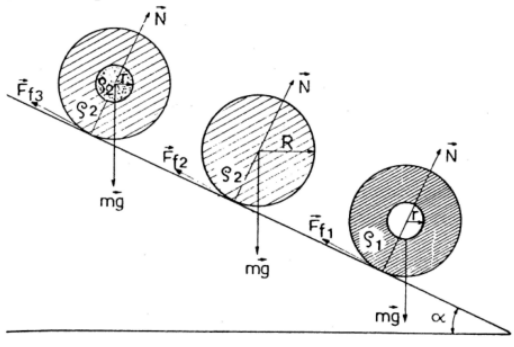


Fig. 1.1

In the case of the cylinders from this problem, the condition necessary so that none of them slides is obtained for maximum I:

$$tg\alpha\langle\mu\left(1+\frac{mR^2}{I_1}\right)=\mu\frac{4n-1}{2n-1} \tag{11}$$

The accelerations of the cylinders are:

$$a_1=\frac{2g\sin\alpha}{3+(1-\frac{1}{n})}\quad,\ a_2=\frac{2g\sin\alpha}{3}\ ,\quad a_3=\frac{2g\sin\alpha}{3-(1-\frac{1}{n})^2}. \tag{12}$$

The relation between accelerations:

$$\mathrm{a_1}<\mathrm{a_2}<\mathrm{a_3} \tag{13}$$

In the case than all the three cylinders slide:

$$F_f=\mu N=\mu mg\cos\alpha \tag{14}$$

and from (7) results:

$$\varepsilon=\frac{R}{I}\mu mg\cos\alpha \tag{15}$$

for the cylinders of the problem:

$$\varepsilon_1:\varepsilon_2:\varepsilon_3=\frac{1}{I_1}:\frac{1}{I_2}:\frac{1}{I_3}=1:\left(1-\frac{1}{n}\right):n$$

$$\varepsilon_1<\varepsilon_2<\ \varepsilon_3 \tag{16}$$

In the case that one of the cylinders is sliding:

$$mg\sin\alpha-F_f=ma\ ,\quad F_f=\mu mg\cos\alpha\ , \tag{17}$$

$$a=g\left(\sin\alpha-\mu\cos\alpha\right) \tag{18}$$

Let $\vec{F}$ be the total force acting on the liquid mass $\mathrm{m_l}$ inside the cylinder (fig.1.2), we can write:

$$F_x+m_lg\sin\alpha=m_la=m_lg\left(\sin\alpha-\mu\cos\alpha\right),\quad F_y-m_lg\cos\alpha=0 \tag{19}$$

$$F=\sqrt{F_x^2+F_y^2}=m_lg\cos\alpha\cdot\sqrt{1+\mu^2}=m_lg\frac{\cos\alpha}{\cos\phi} \tag{20}$$

where ϕ is the friction angle ($tg\phi=\mu$).

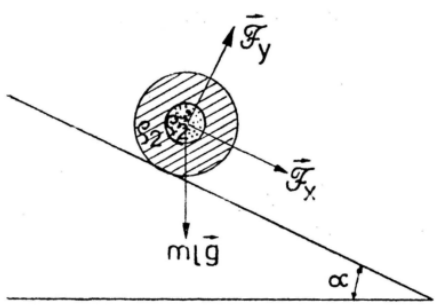


Fig. 1.2