Solution Problem 2

a) We consider argon an ideal mono-atomic gas and the collisions of the atoms with the piston perfect elastic. In such a collision with a fix wall the speed $\vec{v}$ of the particle changes only the direction so that the speed $\vec{v}$ and the speed $\vec{v}\,'$after collision there are in the same plane with the normal and the incident and reflection angle are equal.

$$v_n' = -v_n\,,\ v_t' = v_t \tag{1}$$

In the problem the wall moves with the speed $\vec{u}$ perpendicular on the wall. The relative speed of the particle with respect the wall is $\vec{v}-\vec{u}$. Choosing the Oz axis perpendicular on the wall in the sense of $\vec{u}$, the conditions of the elastic collision give:

$$\begin{gathered}(\vec{v}-\vec{u})_z = -\left(\vec{v}\,'-\vec{u}\right)_z\ ,\ (\vec{v}-\vec{u})_{x,y} = \left(\vec{v}\,'-\vec{u}\right)_{x,y}\ ; \\ v_z - u = -\left(v_z'-u\right)\ ,\ v_z' = 2u - v_z\,,\ v_{x,y}' = v_{x,y}\end{gathered} \tag{2}$$

The increase of the kinetic energy of the particle with mass m_o after collision is:

$$\frac{1}{2}m_o v'^2 - \frac{1}{2}m_o v^2 = \frac{1}{2}m_o\left(v_z'^2 - v_z^2\right) = 2m_o u\left(u - v_z\right) \cong -2m_o u v_z \tag{3}$$

because u is much smaller than v_z .

If n_k is the number of molecules from unit volume with the speed component v_{zk} , then the number of molecules with this component which collide in the time dt the area dS of the piston is:

$$\frac{1}{2}n_k v_{zk} dt dS \tag{4}$$

These molecules will have a change of the kinetic energy:

$$\frac{1}{2}n_k v_{zk} dt dS\left(-2m_o u v_{zk}\right) = -m_o n_k v_{zk}^2 dV \tag{5}$$

where $dV = udtdS$ is the increase of the volume of gas.

The change of the kinetic energy of the gas corresponding to the increase of volume dV is:

$$dE_c = -m_o dV\sum_k n_k v_{zk}^2 = -\frac{1}{3}nm_o\bar{v}^2 dV \tag{6}$$

and:

$$dU = -\frac{2}{3}N\frac{m_o\bar{v}^2}{2}\cdot\frac{dV}{V} = -\frac{2}{3}U\frac{dV}{V} \tag{7}$$

Integrating equation (7) results:

$$UV^{2/3} = const. \tag{8}$$

The internal energy of the ideal mono-atomic gas is proportional with the absolute temperature T and the equation (8) can be written:

$$TV^{2/3} = const. \tag{9}$$

b) The oxygen is in contact with a thermostat and will suffer an isothermal process. The internal energy will be modified only by the adiabatic process suffered by argon gas:

$\Delta U = \nu C_V \Delta T = mc_V \Delta T$ (10)

where ν is the number of kilomoles. For argon $C_V = \frac{3}{2}R$.

For the entire system L=0 and $\Delta U = Q$.

We will use indices 1, respectively 2, for the measures corresponding to argon from cylinder A, respectively oxygen from the cylinder B:

$$\Delta U = \frac{m_1}{\mu_1}\cdot\frac{3}{2}\cdot R\left(T_1' - T_1\right) = Q = \frac{m_1}{\mu_1}\cdot\frac{3}{2}RT_1\left[\left(\frac{V_1}{V_1'}\right)^{2/3} - 1\right] \tag{11}$$

From equation (11) results:

$$T_1 = \frac{2}{3} \cdot \frac{\mu_1}{m_1} \cdot \frac{Q}{R} \cdot \frac{1}{\left(\frac{V_1}{V_1'}\right)^{2/3} - 1} = 1000K \tag{12}$$

$$T_1' = \frac{T_1}{4} = 250K \tag{13}$$

For the isothermal process suffered by oxygen:

$$\frac{\rho_2'}{\rho_2} = \frac{p_2'}{p_2} \tag{14}$$

$p_2' = 2{,}00atm = 2{,}026 \cdot 10^5 N/m^2$

From the equilibrium condition:

$$p_1' = p_2' = 2atm \tag{15}$$

For argon:

$$p_1 = p_1' \cdot \frac{V_1'}{V_1} \cdot \frac{T_1}{T_1'} = 64atm = 64{,}9 \cdot 10^5 N/m^2 \tag{16}$$

$$V_1 = \frac{m_1}{\mu_1} \cdot \frac{RT_1}{p_1} = 1{,}02m^3, V_1' = 8V_1 = 8{,}16m^3 \tag{17}$$

c) When the valve is opened the gases intermix and at thermal equilibrium the final pressure will be p' and the temperature T. The total number of kilomoles is constant:

$$\nu_1 + \nu_2 = \nu' \,, \frac{p_1' V_1'}{RT_1'} + \frac{p_2' V_2'}{RT} = \frac{p\left(V_1' + V_2'\right)}{RT} \tag{18}$$

$$p_1' + p_2' = 2atm, T_2 = T_2' = T = 300K$$

The total volume of the system is constant:

$$V_1 + V_2 = V_1' + V_2', \quad \frac{V_2'}{V_2} = \frac{\rho_2}{\rho_2'}, \quad V_2' = \frac{V_2}{2} = 7{,}14m^3 \tag{19}$$

From equation (18) results the final pressure:

$$p = p_1' \cdot \frac{1}{V_1 + V_2} \cdot \left(V_1' \cdot \frac{T}{T_1'} + V_2'\right) = 2{,}2atm = 2{,}23 \cdot 10^5 N/m^2 \tag{20}$$