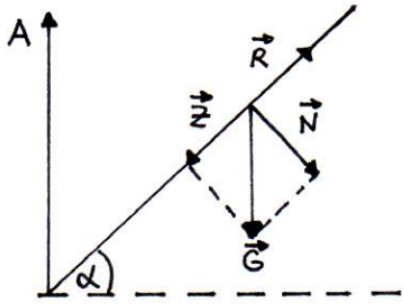
Solution of problem 1:

a) $\omega = 0$:



The forces in this case are (see figure):

$$\vec{G} = \vec{Z} + \vec{N} = m \cdot \vec{g} \tag{1},$$

$$\left|\vec{Z}\right| = m \cdot g \cdot \sin\alpha = Z \tag{2},$$

$$\left|\vec{N}\right| = m \cdot g \cdot \cos\alpha = N \tag{3},$$

$$\left|\vec{R}\right| = \mu \cdot N = \mu \cdot m \cdot g \cdot \cos\alpha = R \tag{4}.$$

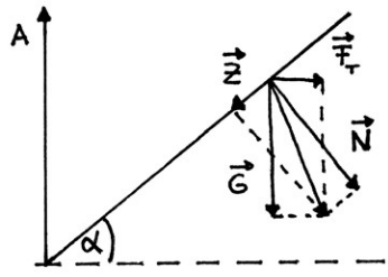
[$\vec{R}$: force of friction]

The body is at rest relative to the rod, if $Z \leq R$. According to equations (2) and (4) this is equivalent to $\tan\alpha \leq \tan\beta$. That means, the body is at rest relative to the rod for $\alpha \leq \beta$ and the body moves along the rod for $\alpha > \beta$.

b) $\omega > 0$:

Two different situations have to be considered: 1. $\alpha > \beta$ and 2. $\alpha \leq \beta$.

If the rod is moving ($\omega \neq 0$) the forces are $\vec{G} = m \cdot \vec{g}$ and $\left|\vec{F}_r\right| = m \cdot r \cdot \vec{\omega}^2$.



From the parallelogramm of forces (see figure):

$$\vec{Z} + \vec{N} = \vec{G} + \vec{F}_r \tag{5}.$$

The condition of equilibrium is:

$$\left|\vec{Z}\right| = \mu \left|\vec{N}\right| \tag{6}.$$

Case 1: $\vec{Z}$ is oriented downwards, i.e. $g \cdot \sin\alpha > r \cdot \omega^2 \cdot \cos\alpha$.

$$\left|\vec{Z}\right| = m \cdot g \cdot \sin\alpha - m \cdot r \cdot \omega^2 \cdot \cos\alpha \text{ and } \left|\vec{N}\right| = m \cdot g \cdot \cos\alpha + m \cdot r \cdot \omega^2 \cdot \sin\alpha$$

Case 2: $\vec{Z}$ is oriented upwards, i.e. $g \cdot \sin\alpha < r \cdot \omega^2 \cdot \cos\alpha$.

$$\left|\vec{Z}\right| = -m \cdot g \cdot \sin\alpha + m \cdot r \cdot \omega^2 \cdot \cos\alpha \text{ and } \left|\vec{N}\right| = m \cdot g \cdot \cos\alpha + m \cdot r \cdot \omega^2 \cdot \sin\alpha$$

It follows from the condition of equilibrium equation (6) that

$$\pm\left(g \cdot \sin\alpha - r \cdot \omega^2 \cdot \cos\alpha\right) = \tan\beta \cdot \left(g \cdot \cos\alpha + r \cdot \omega^2 \cdot \sin\alpha\right) \tag{7}.$$

Algebraic manipulation of equation (7) leads to:

$$g \cdot \sin(\alpha - \beta) = r \cdot \omega^2 \cdot \cos(\alpha - \beta) \quad (8),$$

$$g \cdot \sin(\alpha + \beta) = r \cdot \omega^2 \cdot \cos(\alpha + \beta) \quad (9).$$

That means,

$$r_{1,2} = \frac{g}{\omega^2} \cdot \tan(\alpha \mp \beta) \quad (10).$$

The body is at rest relative to the rotating rod in the case $\alpha > \beta$ if the following inequalities hold:

$$r_1 \leq r \leq r_2 \qquad \text{with } r_1, r_2 > 0 \quad (11)$$

or

$$L_1 \leq L \leq L_2 \qquad \text{with } L_1 = r_1 \,/\, \cos\alpha \text{ and } L_2 = r_2 \,/\, \cos\alpha \quad (12).$$

The body is at rest relative to the rotating rod in the case $\alpha \leq \beta$ if the following inequalities hold:

$$0 \leq r \leq r_2 \qquad \text{with } r_1 = 0 \text{ (since } r_1 < 0 \text{ is not a physical solution), } r_2 > 0 \quad (13).$$

Inequality (13) is equivalent to

$$0 \leq L \leq L_2 \qquad \text{with } L_2 = r_2 \,/\, \cos\alpha > 0 \quad (14).$$

Marking scheme

Problem 1: "Rotating rod" (10 points)

Part a	1 point
Part b – cases 1. and 2.	1 point
– forces and condition of equilibrium	1 point
– case Z downwards	2 points
– case Z upwards	2 points
– calculation of $r_{1,2}$	1 point
– case $\alpha > \beta$	1 point
– case $\alpha \leq \beta$	1 point