The plot of the angle as a function of time for a typical measurement of the acceleration of the disk is shown in Fig. 9.

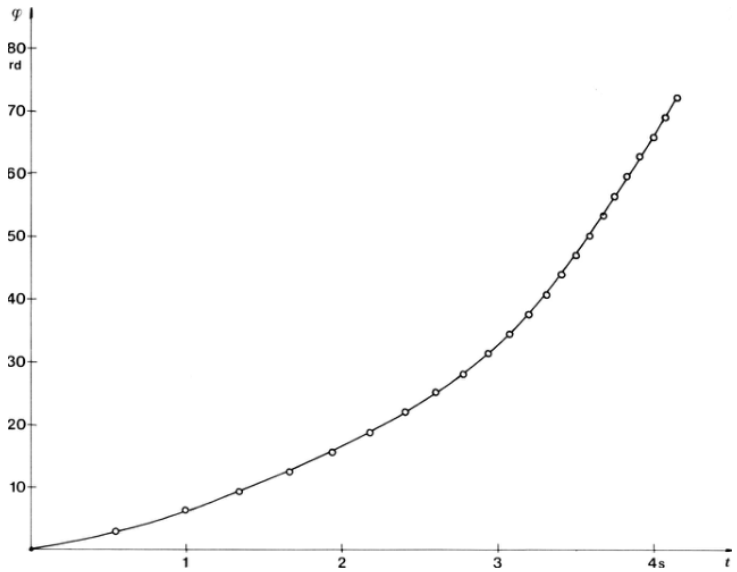


Fig. 9 Angle vs. time

The angular velocity is calculated using the formula:

$$\omega_i(t_i') = \frac{\pi}{(t_{i+i} - t_i)}$$

and corresponds to the time in the middle of the interval (t_i, t_{i+1}): $t_i' = \frac{1}{2}(t_{i+1} + t_i)$. The calculated values are displayed in Table 1 and plotted in Fig. 10.

Observing the time intervals of half turns when the constant angular velocity is reached, one can conclude that the iron pegs are not positioned perfectly symmetrically. This systematic error can be neglected in the calculation of angular velocity, but not in the calculation of angular acceleration. To avoid this error we use the time intervals of full turns:

$$\alpha_i(t_i'') = \frac{\Delta\omega_i}{\Delta t_i}\,,$$

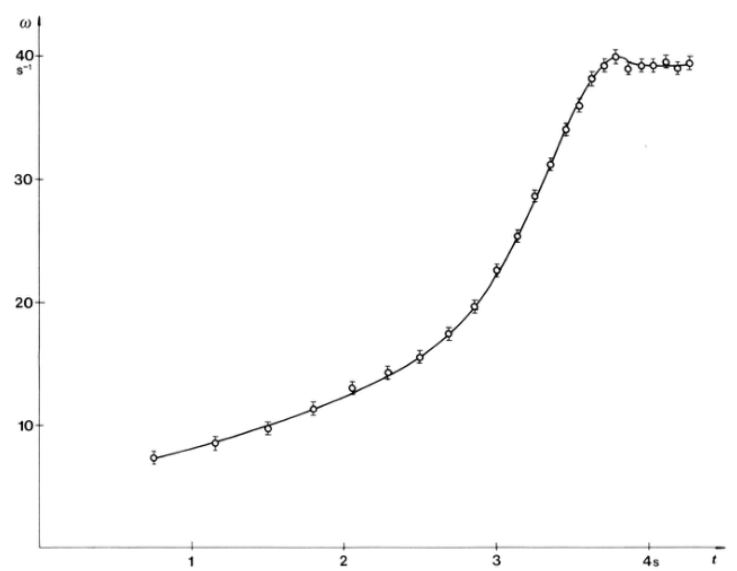


Fig. 10 Angular velocity vs. time

where $\Delta t_i = t_{2i+2} - t_{2i}$,

$$\Delta\omega_i = \frac{2\pi}{(t_{2i+3} - t_{2i+1})} - \frac{2\pi}{(t_{2i+1} - t_{2i-1})}$$

and $t_i'' = t_{2i+1}'$.

The angular acceleration as a function of time is plotted in Fig. 11.

The torque, M, and the power, P, necessary to drive the disk (net torque and net power), are calculated using the relation:

$$M(t) = I\alpha(t)$$

and

$$P(t) = M(t)\omega(t)$$

where the moment of inertia, $I = (14.0 \pm 0.5) \cdot 10^{-6}\,\mathrm{kgm}^2$, is given. The corresponding angular velocity is determined from the plot in Fig. 10 by interpolation. This plot is used also to find the torque and the power as functions of angular velocity (Fig. 12 and 13).

i	t ms	δt ms	φ rd	t' ms	ω s^{-1}	α s^{-2}
1	0.0		0.0			
				272.0	5.78	
2	543.9	543.9	3.14			
				758.7	7.31	
3	973.5	429.6	6.28			3.38
				1156.3	8.60	
4	1339.0	365.5	9.42			
				1499.9	9.76	
5	1660.8	327.8	12.57			5.04
				1798.6	11.40	
6	1936.3	275.5	15.71			
				2057.1	13.01	
7	2177.8	241.5	18.85			5.96
				2287.2	14.36	
8	2396.6	218.8	21.99			
				2498.1	15.48	
9	2599.6	203.0	25.73			9.40
				2689.6	17.46	
10	2779.5	179.9	28.27			
				2859.4	19.66	
11	2939.3	159.8	31.42			18.22
				3008.6	22.65	
12	3078.0	138.7	34.56			
				3139.9	25.38	
13	3201.8	123.8	37.70			25.46
				3256.6	28.66	
14	3311.4	109.6	40.84			
				3361.8	31.20	
15	3472.1	100.7	43.98			26.89
				3458.2	34.11	
16	3504.2	92.1	47.12			
				3547.8	36.07	
17	3591.3	87.1	50.27			21.72
				3632.4	38.27	
18	3673.4	82.1	53.41			
				3713.5	39.22	
19	3753.5	80.1	56.55			4.76
				3792.8	39.97	
20	3832.7	78.6	59.69			
				3872.4	39.03	
21	3912.6	80.5	62.83			−1.69
				3952.7	39.22	
22	3992.7	80.1	65.97			
				4032.8	39.22	
23	4072.8	80.1	69.12			0.77
				4112.4	39.67	
24	4152.0	79.2	72.26			
				4192.3	39.03	
25	4232.5	80.5	75.40			−0.15
				4272.4	39.42	
26	4312.3	79.7	78.54			

Table 1

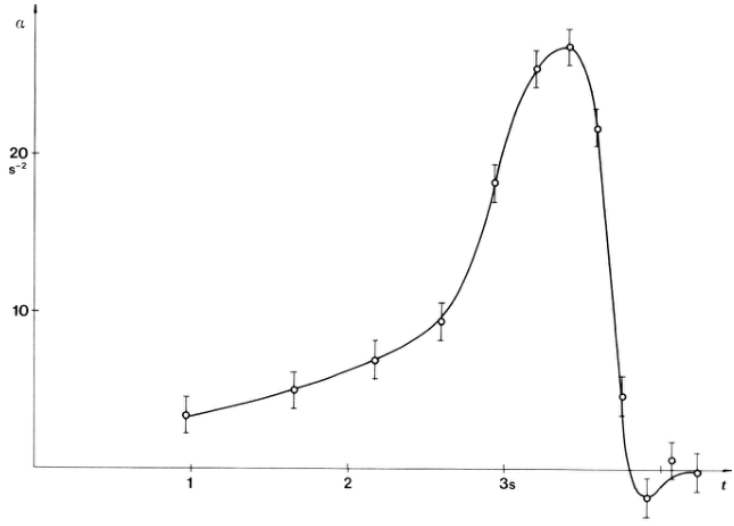


Fig. 11 Angular acceleration vs. time

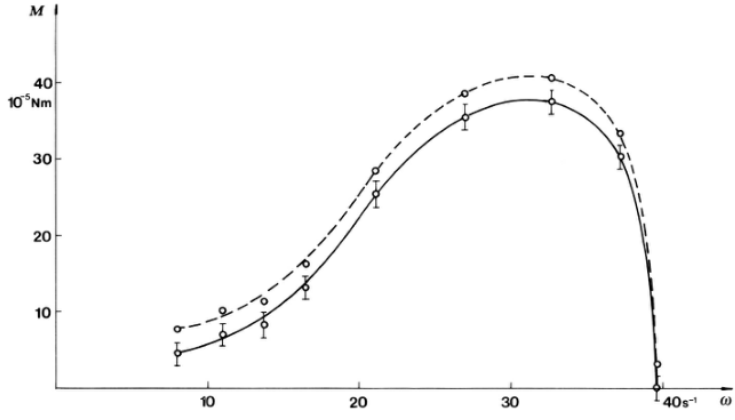


Fig. 12 Net torque (full line) and total torque (dashed line) vs. angular velocity

To find the total torque and the power of the motor, the torque and the power losses due to the friction forces have to be determined and added to the corresponding values of net torque and power. By measuring the angular velocity during the deceleration of the disk after the motor has been switched off (Fig. 14), we can determine the torque of friction which is approximately constant and is equal to $M' = (3.1 \pm 0.3) \cdot 10^{-5}\,\mathrm{Nm}$.

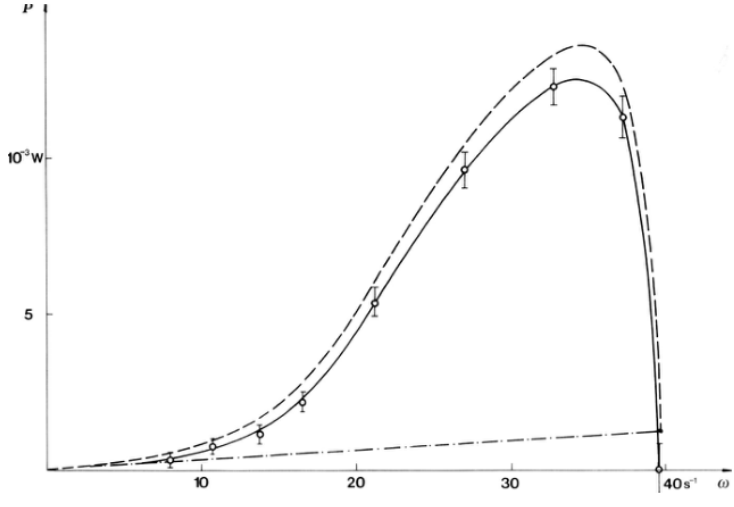


Fig. 13 Net power (full line), power loses (dashed and dotted line) and total power (dashed line) vs. angular velocity

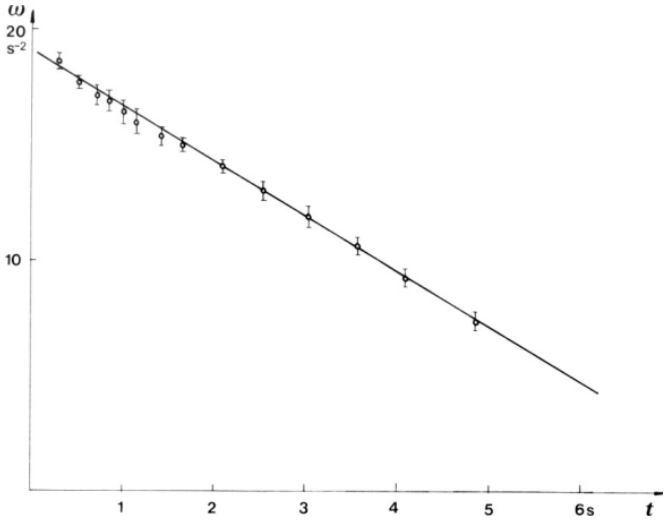


Fig. 14 Angular velocity vs. time during deceleration

The total torque and the total power are shown in Fig. 12 and 13.

Marking scheme

a) Determination of errors 1 p.

b) Plot of angle vs. time 1 p.

c) Plot of angular velocity and acceleration 3 p.

d) Correct times for angular velocity 1 p.

e) Plot of net torque vs. angular velocity 2 p. (Plot of torque vs. time only, 1 p.)

f) Plot of net power vs. angular velocity 1 p.

g) Determination of friction 1 p.