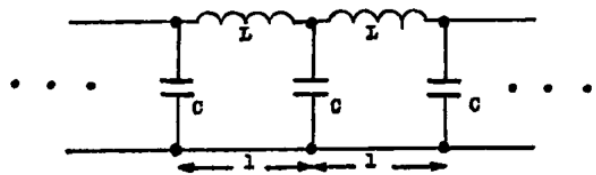
Problem 3: Infinite LC-grid

When sine waves propagate in an infinite LC-grid (see the figure below) the phase of the ac-voltage across two successive capacitors differs by Φ.



a) Determine how Φ depends on ω, L and C (ω is the angular frequency of the sine wave).

b) Determine the velocity of propagation of the waves if the length of each unit is ℓ.

c) State under what conditions the propagation velocity of the waves is almost independent of ω. Determine the velocity in this case.

d) Suggest a simple mechanical model which is an analogue to the above circuit and derive equations which establish the validity of your model.

Formulae:

$$\cos\alpha - \cos\beta = -2 \cdot \sin\left(\frac{\alpha+\beta}{2}\right) \cdot \sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2 \cdot \cos\left(\frac{\alpha+\beta}{2}\right) \cdot \sin\left(\frac{\alpha-\beta}{2}\right)$$