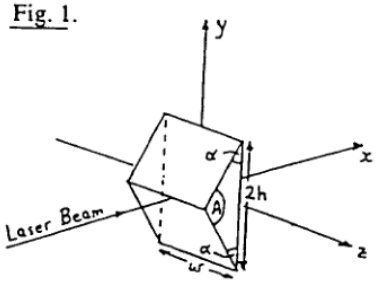
LASER FORCES ON A TRANSPARENT PRISM

By means of refraction a strong laser beam can exert appreciable forces on small transparent objects. To see that this is so, consider a small glass triangular prism with an apex angle $A = \pi$ - 2α, a base of length $2h$ and a width w. The prism has an index of refraction n and a mass density ρ.

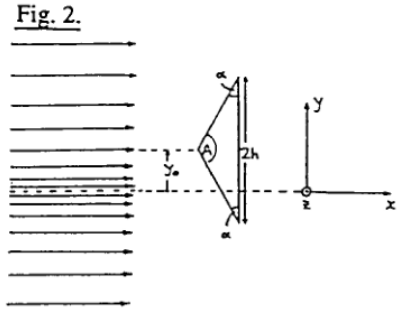
Suppose that this prism is placed in a laser beam travelling horizontally in the x direction. (Throughout this problem assume that the prism does not rotate, i.e., its apex always points opposite to the direction of the laser beam, its triangular faces are parallel to the xy plane, and its base is parallel to the yz plane, as shown in Fig. 1.) Take the index of refraction of the surrounding air to be $n_{\text{air}} = 1$. Assume that the faces of the prism are coated with an anti-reflection coating so that no reflection occurs.

Fig. 1.



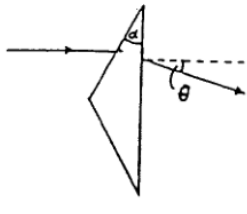
The laser beam has an intensity that is uniform across its width in the z direction but falls off linearly with distance y from the x axis such that it has a maximum value of I_0 at $y = 0$ and falls to zero at $y = \pm 4h$ (Fig. 2). [Intensity is power per unit area, e.g. expressed in W m^{-2}.]

Fig. 2.



1) Write equations from which the angle θ (see Fig. 3) may be determined (in terms of α and n) in the case when laser light strikes the upper face of the prism.

Fig. 3.



2) Express, in terms of I_0, θ, h, w and y_0, the x and y components of the net force exerted on the prism by the laser light when the apex of the prism is displaced a distance y_0 from the x axis where $|y_0| \leq 3h$.

Plot graphs of the values of the horizontal and vertical components of force as functions of vertical displacement y_0.

3) Suppose that the laser beam is 1 mm wide in the z direction and 80 μm thick (in the y direction). The prism has $\alpha = 30^{\text{o}}$, $h = 10$ μm, $n = 1.5$, $w = 1$ mm and $\rho = 2.5$ g

cm^{-3}. How many watts of laser power would be required to balance this prism against the pull of gravity (in the -y direction) when the apex of the prism is at a distance $y_0 = -h/2$ (= -5 μm) below the axis of the laser beam?

4) Suppose that this experiment is done in the absence of gravity with the same prism and a laser beam with the same dimensions as in (3), but with $I_0 = 10^8$ W m^{-2}.

What would be the period of oscillations that occur when the prism is displaced and released a distance $y = h/20$ from the center line of the laser beam?