Theoretical Problem 2—Solution

1) For t=t_1 to t_3

Since $r = 0$, the voltage across the magnet $V_M = LdI_1 / dt$ =0, therefore,

$$I_1 = I_1(t_1) = \frac{1}{2} I_0 ;$$
$$I_2 = I - I_1 = I - \frac{1}{2} I_0 .$$

For t=t_3 to t_4

Since I_2=0 at t=t_3, and I is kept at $\frac{1}{2} I_0$ after $t = t_3$, $V_M = I_2 r_n = 0$, therefore, I_1 and I_2 will not change.

$$I_1 = \frac{1}{2} I_0 ;$$
$$I_2 = 0$$

These results are shown in Fig. 6.

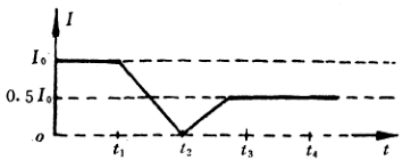
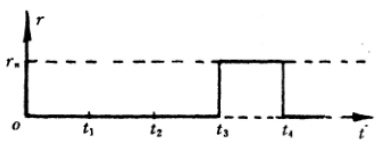
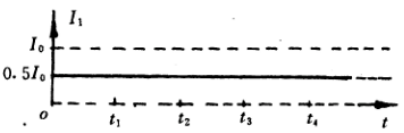


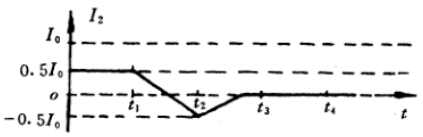
Fig. 6a



6b



6c



6d

2) For $t = 0$ to $t = 1$ min:

Since $r = 0$, $V_M = LdI_1 / dt = 0$

$$I_1 = I_1(0) = 0$$
$$I_2 = I - I_1 = 0.5\,\text{A}.$$

At $t = 1$ min, r suddenly jumps from O to r_n, I will drop from E / R to $E /(R + r_n)$ instantaneously, because I_1 can not change abruptly due to the inductance of the magnet coil. For E / R=0.5A, $R = 7.5\Omega$ and $R_n = 5\Omega$. I will drop to 0.3A.

For $t = 1$ min to 2 min:

I, I_1 and I_2 gradually approach their steady values:

$$I = \frac{E}{R} = 0.5\,\text{A},$$
$$I_1 = I = 0.5\,\text{A}$$
$$I_2 = 0 .$$

The time constant

$$\tau = \frac{L(R + r_n)}{R r_n} .$$

When $L = 10\,\mathrm{H}$, $R = 7.5\Omega$ and $r_n = 5\Omega$, $\tau = 3\,\mathrm{sec}$.

For $t = 2\,\mathrm{min}$ to 3 min:

Since $r = 0$, I_1 and I_2 will not change, that is

$$I_1 = 0.5\,\mathrm{A} \text{ and } I_2 = 0$$

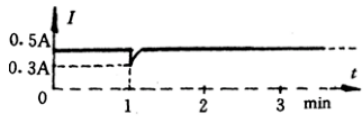
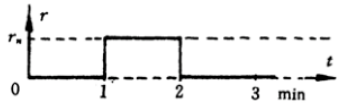
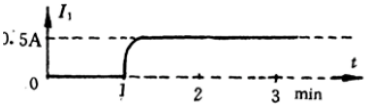


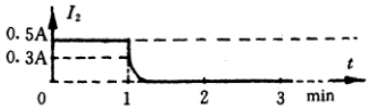
Fig. 7a



7b



7c



7d

3) The operation steps are:

First step

Turn on power switch K, and increase the total current I to 20 A, i. e. equal to I_1. Since the superconducting switch is in the state $r = 0$, so that $V_M = L\ dI_1 / dt = 0$, that is, I_1 can not change and I_2 increases by 20A, in other words, I_2 changes from $-20\,\mathrm{A}$ to zero.

Second step

Switch r from 0 to r_n.

Third step

Gradually reduce I to zero while keeping $I_2 < 0.5\,\mathrm{A}$: since $I_2 = V_M / r_n$ and $V_m = L\,dI_1 / dt$, when $L = 10\,\mathrm{H}$, $r_n = 5\Omega$, the requirement $I_2 < 0.5\,\mathrm{A}$ corresponds to $dI_1 / dt < 0.25\,\mathrm{A/sec}$, that is, a drop of <15A in 1 min. In Fig. 8 $dI / dt \sim 0.1\mathrm{A/sec}$ and dI_1 / dt is around this value too, so the requirement has been fulfilled.

Final step

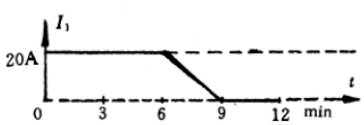
Switch r to zero when $V_M = 0$ and turn off the power switch K. These results are shown in Fig. 8.



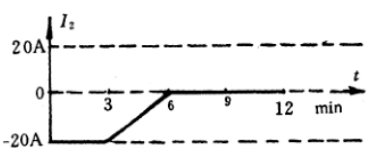
Fig. 8a



8b



8c



8d

4) **First step** and **second step** are the same as that in part 3, resulting in $I_2 = 0$.

Third step Increase I by 10 A to 30 A with a rate subject to the requirement $I_2 < 0.5$ A.

Fourth step Switch r to zero when $V_M = 0$.

Fifth step Reduce I to zero, $I_1 = 30$ A will not change because V_M is zero. $I_2 = I - I_1$ will change to -30 A. The current flowing through the magnet is thus closed by the superconducting switch.

Final step Turn off the power switch K. The magnet is operating in the persistent mode.

These results are shown in Fig. 9.

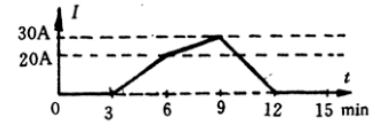
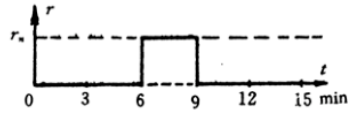
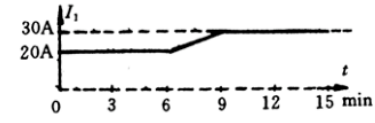


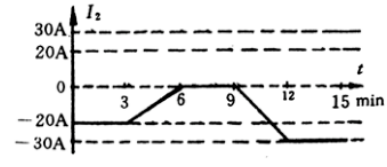
Fig. 9a



9b



9c



9d

Grading Scheme

Part 1, 2 points:

0.5 point for each of I_1, I_2 from $t = t_1$ to t_3 and I_1, I_2 from $t = t_3$ to t_4.

Part 2, 3 points:

0.3 point for each of I_1, I_2 from $t = 0$ to 1 min, I, I_1, I_2 at $t = 1$ min, and I_0, I_1, I_2 from $t = 1$ to 2 min;

0.2 point for each of I, I_1, and I_2 from $t = 2$ to 3 min.

Part 3, 2 points:

0.25 point for each section in Fig. 8 from $t = 3$ to 9 min, 8 sections in total.

Part 4, 3 points:

0.25 point for each section in Fig. 9 from $t = 3$ to 12 min, 12 sections in total.