Theory Question No.1

Scaling

(a) A small mass hangs on the end of a massless ideal spring and oscillates up and down at its natural frequency f. If the spring is cut in half and the mass reattached at the end, what is the new frequency f' ? (1.5 *mark*s)

(b) The radius of a hydrogen atom in its ground state is $a_0 = 0.0529\,\text{nm}$ (the "Bohr radius"). What is the radius a' of a "muonic-hydrogen" atom in which the electron is replaced by an identically charged muon, with mass 207 times that of the electron? Assume the proton mass is much larger than that of the muon and electron. (2 *marks*)

(c) The mean temperature of the earth is $T = 287\,\text{K}$. What would the new mean temperature T' be if the mean distance between the earth and the sun was reduced by 1%?

(2 *marks*)

(d) On a given day, the air is dry and has a density $\rho = 1.2500\,\text{kg/m}^3$. The next day the humidity has increased and the air is 2% by mass water vapour. The pressure and temperature are the same as the day before. What is the air density ρ' now? (2 *marks*)

Mean molecular weight of dry air: 28.8 (g/mol)
Molecular weight of water: 18 (g/mol)

Assume ideal-gas behaviour.

(e) A type of helicopter can hover if the mechanical power output of its engine is P. If another helicopter is made which is an exact ½-scale replica (in all linear dimensions) of the first, what mechanical power P' is required for it to hover? (*2.5 marks*)