MARK SCHEME AND SOLUTIONS FOR Q3

Total marks = 10

A **a)** $\Delta x_t = ae^{-\mu t}\cos(\omega t + \phi),\ 0.8 = e^{-50\mu} \Rightarrow \mu = 4.5 \times 10^{-3}\,\mathrm{s}^{-1}$. [0.1]

b) $\upsilon = (E/\rho)^{1/2} = (7.1 \times 10^{10}/2700)^{1/2} = 5100\,\mathrm{m.s}^{-1}$.

At fundamental $\lambda_{rod} = 4l = 4\,\mathrm{m}$.

$f = 5100/4 = 1.3 \times 10^3\,\mathrm{Hz}$.

$\omega = 2\pi f = 8.1 \times 10^3\,\mathrm{rad.s}^{-1}$. [0.1]

c) $\upsilon = f\lambda_{rod},\ \delta\lambda_{rod}/\lambda_{rod} = (-)\delta f/f \Rightarrow \delta l/l$. [0.8]

$\delta l = l.(\delta f/f)$. [0.6]

$\delta l = 1 \times (5.0 \times 10^{-3}/1.3 \times 10^3) = 3.8 \times 10^{-6}\,\mathrm{m}$. [0.1]

d) Change in gravitational force on rod at a distance x from the free end $= m\Delta g$ and $m = \rho x A$,

where A is the cross-sectional area of the rod. [0.5]

Change in stress $= m\Delta g/A = \rho x \Delta g$. [0.5]

Change in strain $= \delta(dx)/dx = \rho x \Delta g/E$;

that is, $dx \rightarrow (1 + \rho x \Delta g/E)\,dx \Rightarrow \Delta l = (\rho \Delta g/2E)\,l^2$. [0.5]

e) At fundamental $\lambda_{rod} = 4l \Rightarrow \Delta l = \Delta\lambda_{rod}/4$,

for $\Delta\lambda_{rod} = 656\,\mathrm{nm}/10^4 \Rightarrow \Delta l = 656\,\mathrm{nm}/(4 \times 10^4)$. [0.1]

$\Delta l = 656\,\mathrm{nm}/(4 \times 10^4) = (\rho \Delta g/2E)\,l^2$ [0.1]

$\Delta l = (2700 \times 10^{-19}/14 \times 10^{10})\,l^2 \Rightarrow l = 9.2 \times 10^7\,\mathrm{m}$. [0.1]

B **a)** $mc^2 = hf \Rightarrow m = hf/c^2$, [0.3]

$hf' = hf - GMm/R$, [0.3]

$\Rightarrow hf' = hf\,(1 - GM/Rc^2), \therefore f' = f\,(1 - GM/Rc^2)$. [0.4]

b) $n_r = c/c\,(1 - GM/rc^2)^2$, [1.0]

$n_r = 1 + 2GM/rc^2$, for small GM/rc^2; i.e. $\alpha = 2$. [1.0]

c)

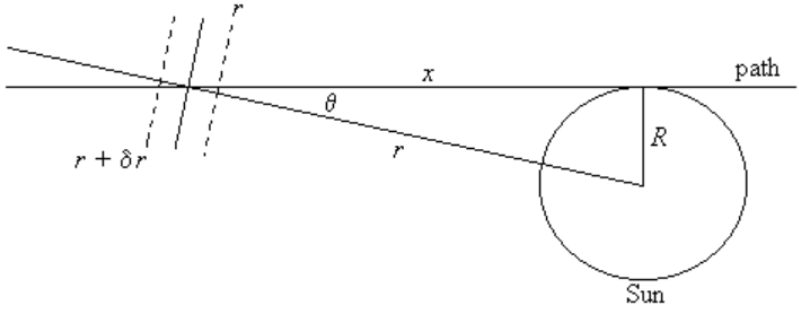


Diagram [0.2]

By Snell's law: $n(r + \delta r)\sin\theta = n(r)\sin(\theta - \delta\xi)$, [1.0]

$(n(r) + (dn/dr)\,\delta r)\sin\theta = n(r)\sin\theta - n(r)\cos\theta\,\delta\xi$. [0.4]

$(dn/dr)\,\delta r \sin\theta = -n(r)\cos\theta\,\delta\xi$.

Now $n(r) = 1 + 2GM/rc^2$, so $(dn/dr) = -2GM/c^2r^2$, [0.3]

and $(2GM/c^2r^2)\sin\theta\,\delta r = n(r)\cos\theta\,\delta\xi$.

Hence $\delta\xi = (2GM/c^2r^2)\tan\theta\,(\delta r/n) \approx (2GM\tan\theta/c^2r^2)\,\delta r.$ [1.0]

Now $r^2 = x^2 + R^2$, so $r\,dr = x\,dx.$ [0.1]

$$\int d\xi = \frac{2GM}{c^2}\int\frac{\tan\theta\,dr}{r^2} = \frac{2GM}{c^2}\int\frac{\tan\theta\,r\,dr}{r^3} = \frac{2GMR}{c^2}\int_{-\infty}^{+\infty}\frac{dx}{(x^2+R^2)^{3/2}}$$

$$\xi = \frac{4GM}{Rc^2}\,radians = 8.4\times10^{-6}\,radians.$$

[0.5]