**Part 2a**

The total energy radiated per second = $4\pi R^2\sigma T^4$, where σ is the Stephan-Boltzmann constant. The energy incident on a unit area on earth per second is;

$$P = \frac{4\pi R^2\sigma T^4}{4\pi\ell^2} \quad \text{yielding,} \quad R = \left(P/\sigma T^4\right)^{1/2}\ell \tag{1}$$

(0.8 pts)

The energy of a photon is hf = hc/λ. The equivalent mass of a photon is h/cλ. Conservation of photon energy:

$$\frac{hc}{\lambda_0} - \frac{Gm_0}{R}\cdot\frac{h}{c\lambda_0} = \frac{hc}{\lambda}$$

(0.8 pts)

yielding

$$R = \frac{Gm_0\left(\lambda_0 + \Delta\lambda\right)}{c^2\Delta\lambda} \tag{2}$$

and (2) yields,

$$m_0 = \frac{c^2\Delta\lambda\left(P/\sigma T^4\right)^{1/2}}{G\left(\lambda_0 + \Delta\lambda\right)}\ell \tag{3}$$

(0.2 pts)

The stars are rotating around the center of mass with equal angular speeds:

$$\omega = (2\pi/2\tau) = \pi/\tau \tag{4}$$

(0.2 pts)

The equilibrium conditions for the stars are;

$$\frac{GMm_0}{\left(r_1 + r_2\right)^2} = m_0 r_1\omega^2 = Mr_2\omega^2 \tag{5}$$

(0.8 pts)

with

$$r_1 = \ell\frac{\Delta\theta}{2}, \quad r_2 = \ell\frac{\Delta\phi}{2} \tag{6}$$

(0.4 pts)

Substituting (3), (4) and (6) into (5) yields

$$\ell = \left(\frac{8c^2\Delta\lambda\left(P/\sigma T^4\right)^{1/2}}{\Delta\phi\left(\pi/\tau\right)^2\left(\lambda_0 + \Delta\lambda\right)\left(\Delta\theta + \Delta\phi\right)^2}\right)^{1/2}.$$

(0.8 pts)

**Part 2b**

Conservation of angular momentum for the ordinary star;

$$mr^2\omega = m_0 r_0^2\omega_0 \tag{7}$$

(0.6 pts.)

Conservation of angular momentum for dm:

$$r^2\omega\, dm = r_f^2\omega_f\, dm \tag{8}$$

(0.6 pts)

where $\omega_f$ is the angular velocity of the ring. Equilibrium in the original state yields,

$$\omega_0 = \left(\frac{GM}{r_0^3}\right)^{1/2} \tag{9}$$

(0.8 pts)

and (7), (8) and (9) give,

$$\omega = \frac{m_0 r_0}{mr^2}\left(\frac{GM}{r_0}\right)^{1/2}, \quad \omega_f = \frac{m_0 r_0}{mr_f^2}\left(\frac{GM}{r_0}\right)^{1/2} \tag{10}$$

(0.4 pts)

Conservation of energy for dm;

$$\frac{1}{2}\,dm\left(v_0^2 + r^2\omega^2\right) - \frac{GM\,\mathrm{dm}}{\mathrm{r}} = \frac{1}{2}\,dm\,r_f^2\omega_f^2 - \frac{GM\,\mathrm{dm}}{\mathrm{r_f}} \tag{11}$$

(1.2 pts)

Substituting (10);

$$v_0^2 + \frac{m_0^2 r_0 GM}{m^2}\left(\frac{1}{r^2} - \frac{1}{r_f^2}\right) - 2GM\left(\frac{1}{r} - \frac{1}{r_f}\right) = 0 \tag{12}$$

Since $\mathrm{r_0} >> \mathrm{r_f}$, if $\mathrm{r} > \mathrm{r_0}$, $\mathrm{r}^{-1}$ and $\mathrm{r}^{-2}$ terms can be neglected. Hence,

$$r_f = \frac{GM}{\nu_0^2}\left(\left(1 + \frac{m_0^2 r_0 \nu_0^2}{GMm^2}\right)^{1/2} - 1\right).$$

(0.8 pts)

To show that $\mathrm{r} > \mathrm{r_0}$ change in the linear momentum of the ordinary star in its reference frame:

$$-\frac{GMm}{r^2} + mr\omega^2 - m\frac{dv_r}{dt} = -v_0\frac{dm_{gas}}{dt} \tag{13}$$

(0.8 pts)

and (13) implies the existence of an outward force initially and hence r starts growing. Using (7) one can write

$$mr\omega^2 = \frac{m_0^2 r_0^4 \omega_0^2}{mr^3}\,.$$

Hence, $\dfrac{\textit{Gravitational force}}{\textit{Centrifugal force}} \,\alpha\, \mathrm{m}^2 r\,.$ (0.4 pts)

where m is definitely decreasing. If r starts decreasing at some time also, this ratio starts decreasing, which is a contradiction.

So $\mathrm{r} > \mathrm{r_0}$. (0.4 pts)