Solution I: Ground-Penetrating Radar

1. Speed of radar signal in the material v_m:

$$\omega t - \beta z = \text{constant} \rightarrow \beta z = -\text{constant} + \omega t$$

(0.2 pts)

$$v_m = \frac{\omega}{\beta}$$

$$v_m = \frac{1}{\omega \left\{ \frac{\mu\varepsilon}{2} \left[\left(1 + \frac{\sigma^2}{\varepsilon^2\omega^2} \right)^{1/2} + 1 \right] \right\}^{1/2}}$$

(0.4 pts)

$$v_m = \frac{1}{\left\{ \frac{\mu\varepsilon}{2}(1+1) \right\}^{1/2}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

(0.4 pts)

2. The maximum depth of detection (skin depth, δ) of an object in the ground is inversely proportional to the attenuation constant: **(0.5 pts) (0.3 pts) (0.2 pts)**

$$\delta = \frac{1}{a} = \frac{1}{\omega \left\{ \frac{\mu\varepsilon}{2} \left[\left(1 + \frac{\sigma^2}{\varepsilon^2\omega^2} \right)^{1/2} - 1 \right] \right\}^{1/2}} = \frac{1}{\omega \left\{ \frac{\mu\varepsilon}{2} \left[\left(1 + \frac{1}{2}\frac{\sigma^2}{\varepsilon^2\omega^2} \right) - 1 \right] \right\}^{1/2}} = \frac{1}{\omega \left\{ \frac{\mu\varepsilon}{2} \cdot \frac{1}{2}\frac{\sigma^2}{\varepsilon^2\omega} \right.}$$

$$\delta = \left(\frac{2}{\sigma} \right) \left(\frac{\varepsilon}{\mu} \right)^{1/2}.$$

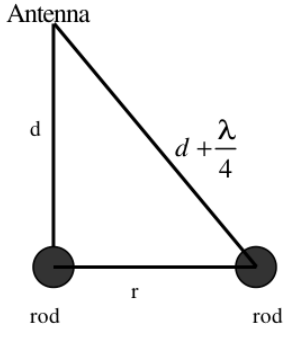
Numerically $\delta = \frac{(5.31\sqrt{\varepsilon_r})}{\sigma}$ m, where σ is in mS/m. **(0.5 pts)**

For a medium with conductivity of 1.0 mS/m and relative permittivity of 9, the skin depth

$$\delta = \frac{(5.31\sqrt{9})}{1.0} = 15.93 \text{ m}$$

(0.3 pts) + (0.2 pts)

3. Lateral resolution:



$$r^2 + d^2 = \left(d + \frac{\lambda}{4} \right)^2$$

$$r = \left(\frac{\lambda d}{2} + \frac{\lambda^2}{16} \right)^{1/2}$$

(1.0 pts)

$r = 0.5$ m, $d = 4$ m: $\quad \frac{1}{2} = \left(\frac{4\lambda}{2} + \frac{\lambda^2}{16} \right)^{1/2}, \quad \lambda^2 + 32\lambda - 4 = 0$ **(0.5 pts)**

The wavelength is $\lambda = 0.125$ m. **(0.3 pts) + (0.2 pts)**

The propagation speed of the signal in medium is

$$v_m = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r}} = \frac{1}{\sqrt{\mu_o\varepsilon_o}}\frac{1}{\sqrt{\mu_r\varepsilon_r}}$$

$$v_m = \frac{c}{\sqrt{\mu_r\varepsilon_r}} = \frac{0.3}{\sqrt{\varepsilon_r}} \text{ m/ns}, \quad \text{where } c = \frac{1}{\sqrt{\mu_o\varepsilon_o}} \text{ and } \mu_r = 1$$

$$v_m = 0.1 \text{ m/ns} = 10^8 \text{ m/s}$$

(0.5 pts)

The minimum frequency need to distinguish the two rods as two separate objects is

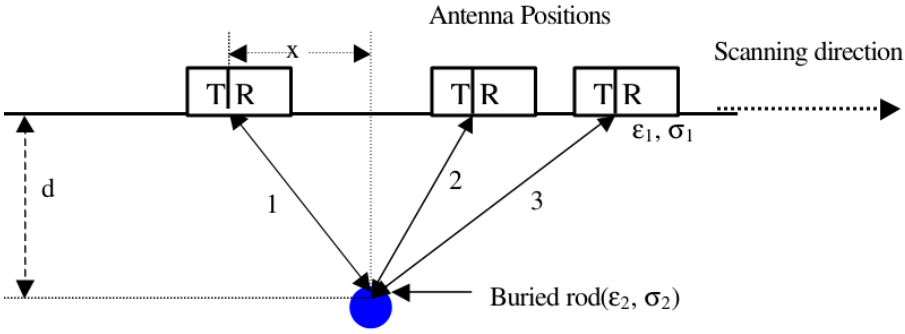
$$f_{\min} = \frac{v}{\lambda}$$

(0.5 pts)

$$f_{\min} = \frac{\frac{0.3}{\sqrt{9}}}{0.125} x10^9 \text{ Hz} = 800 \text{ MHz}$$

(0.3 pts) + (0.20 pts)

4. Path of EM waves for some positions on the ground surface



The traveltime as function of x is

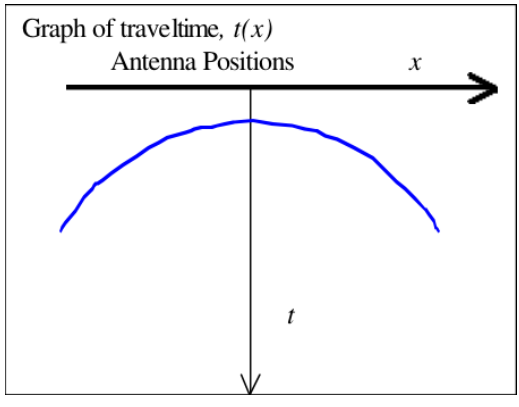
$$\left(\frac{t\,v}{2}\right)^2 = d^2 + x^2,$$

(1.0 pts)

$$t(x) = \sqrt{\frac{4d^2 + 4x^2}{v}}$$

(1.0 pts)

$$t(x) = \frac{2\sqrt{\varepsilon_{1r}}}{0.3}\sqrt{d^2 + x^2}$$



For $x = 0$ **(1.0 pts)**

$$100 = 2 \times (3/0.3)\, d$$

$$d = 5\,\text{m}$$

(0.5 pts)