Theoretical Question 1

A Swing with a Falling Weight

A rigid cylindrical rod of radius R is held horizontal above the ground. With a string of negligible mass and length L ($L > 2\pi R$), a pendulum bob of mass m is suspended from point A at the top of the rod as shown in Figure 1a. The bob is raised until it is level with A and then released from rest when the string is taut. Neglect any stretching of the string. Assume the pendulum bob may be treated as a mass point and swings only in a plane perpendicular to the axis of the rod. Accordingly, the pendulum bob is also referred to as the *particle*. The acceleration of gravity is $\vec{g}$.

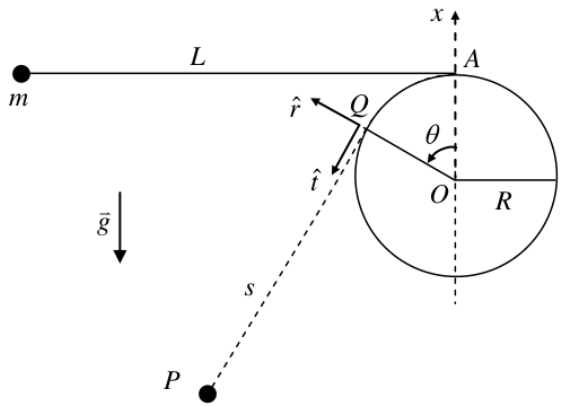


Figure 1a

Let O be the origin of the coordinate system. When the particle is at point P, the string is tangential to the cylindrical surface at Q. The length of the line segment QP is called s. The unit tangent vector and the unit radial vector at Q are given by $\hat{t}$ and $\hat{r}$, respectively. The angular displacement θ of the radius OQ, as measured counterclockwise from the vertical x-axis along OA, is taken to be positive.

When $\theta = 0$, the length s is equal to L and the gravitational potential energy U of the particle is zero. As the particle moves, the instantaneous time rates of change of θ and s are given by $\dot{\theta}$ and $\dot{s}$, respectively.

Unless otherwise stated, all the speeds and velocities are relative to the fixed point O.

Part A

In Part A, the string is taut as the particle moves. In terms of the quantities introduced above (i.e., s, θ, $\dot{s}$, $\dot{\theta}$, R, L, g, $\hat{t}$ and $\hat{r}$), find:

(a) The relation between $\dot{\theta}$ and $\dot{s}$. (0.5 point)

(b) The velocity $\vec{v}_Q$ of the moving point Q relative to O. (0.5 point)

(c) The particle's velocity $\vec{v}\,'$ relative to the moving point Q when it is at P (0.7 point)

(d) The particle's velocity $\vec{v}$ relative to O when it is at P. (0.7 point)

(e) The $\hat{t}$-component of the particle's *acceleration* relative to O when it is at P. (0.7 point)

(f) The particle's gravitational potential energy U when it is at P. (0.5 point)

(g) The speed v_m of the particle at the lowest point of its trajectory. (0.7 point)

Part B

In Part B, the ratio L to R has the following value:

$$\frac{L}{R} = \frac{9\pi}{8} + \frac{2}{3}\cot\frac{\pi}{16} = 3.534 + 3.352 = 6.886$$

(h) What is the speed v_s of the particle when the string segment from Q to P is both straight and shortest in length? (in terms of g and R) (2.4 points)

(i) What is the speed v_H of the particle at its highest point H when it has swung to the other side of the rod? (in terms of g and R) (1.9 points)

Part C

In Part C, instead of being suspended from A, the pendulum bob of mass m is connected by a string over the top of the rod to a heavier weight of mass M, as shown in Figure 1b. The weight can also be treated as a particle.

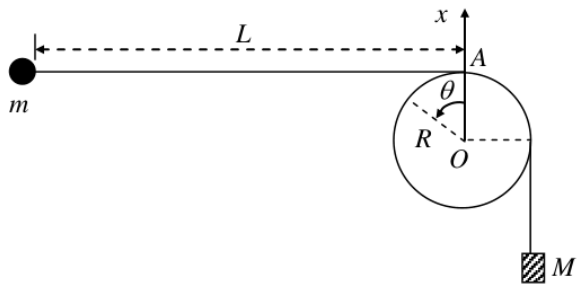


Figure 1b

Initially, the bob is held stationary at the same level as A so that, with the weight hanging below O, the string is taut with a horizontal section of length L. The bob is then released from rest and the weight starts falling. Assume that the bob remains in a vertical plane and can swing past the falling weight without any interruption.

The kinetic friction between the string and the rod surface is negligible. But the static friction is assumed to be large enough so that the weight will remain stationary once it has come to a stop (i.e. zero velocity).

(j) Assume that the weight indeed comes to a stop after falling a distance D and that $(L - D) >> R$. If the particle can then swing around the rod to $\theta = 2\pi$ while both segments of the string free from the rod remain straight, the ratio $\alpha = D/L$ must not be smaller than a critical value α_c. Neglecting terms of the order R/L or higher, obtain an estimate on α_c in terms of M/m. (3.4 points)