Th 2 ABSOLUTE MEASUREMENTS OF ELECTRICAL QUANTITIES

SOLUTION

1. After some time t, the normal to the coil plane makes an angle ωt with the magnetic field $\vec{B}_0 = B_0\vec{i}$. Then, the magnetic flux through the coil is

$$\phi = N\,\vec{B}_0 \cdot \vec{S}$$

where the vector surface $\vec{S}$ is given by $\vec{S} = \pi a^2\left(\cos\omega t\,\vec{i} + \sin\omega t\,\vec{j}\right)$

Therefore $\phi = N\pi a^2 B_0 \cos\omega t$

The induced electromotive force is

$$\mathcal{E} = -\frac{d\phi}{dt} \quad \Rightarrow \quad \mathcal{E} = N\pi a^2 B_0 \omega \sin\omega t$$

The instantaneous power is $P = \mathcal{E}^2/\mathrm{R}$, therefore

$$\langle P\rangle = \frac{\left(N\pi a^2 B_0\omega\right)^2}{2R}$$

where we used $<\sin^2\omega t>=\frac{1}{T}\int_0^T \sin^2\omega t\,dt = \frac{1}{2}$

2. The total field at the center the coil at the instant t is

$$\vec{B}_t = \vec{B}_0 + \vec{B}_i$$

where $\vec{B}_i$ is the magnetic field due to the induced current $\vec{B}_i = B_i\left(\cos\omega t\,\vec{i} + \sin\omega t\,\vec{j}\right)$

with $B_i = \frac{\mu_0 NI}{2a}$ and $I = \mathcal{E}/R$

Therefore $B_i = \frac{\mu_0 N^2\pi a B_0\omega}{2R}\sin\omega t$

The mean values of its components are

$$\langle B_{ix}\rangle = \frac{\mu_0 N^2\pi a B_0\omega}{2R}\langle\sin\omega t\cos\omega t\rangle = 0$$

$$\langle B_{iy}\rangle = \frac{\mu_0 N^2\pi a B_0\omega}{2R}\langle\sin^2\omega t\rangle = \frac{\mu_0 N^2\pi a B_0\omega}{4R}$$

And the mean value of the total magnetic field is

$$\langle\vec{B}_t\rangle = B_0\vec{i} + \frac{\mu_0 N^2\pi a B_0\omega}{4R}\,\vec{j}$$

The needle orients along the mean field, therefore

$$\tan\theta = \frac{\mu_0 N^2\pi a\omega}{4R}$$

Finally, the resistance of the coil measured by this procedure, in terms of θ, is

$$R = \frac{\mu_0 N^2\pi a\omega}{4\tan\theta}$$

3. The force on a unit positive charge in a disk is radial and its modulus is

$$\left|\vec{v}\times\vec{B}\right| = vB = \omega r B$$

where B is the magnetic field at the center of the coil

$$B = N\frac{\mu_0 I}{2a}$$

Then, the electromotive force (e.m.f.) induced on each disk by the magnetic field B is

$$\mathcal{E}_D = \mathcal{E}_{D'} = B\omega \int_0^b r\, dr = \frac{1}{2} B\omega b^2$$

Finally, the induced e.m.f. between 1 and 4 is $\mathcal{E} = \mathcal{E}_D + \mathcal{E}_{D'}$

$$\mathcal{E} = N\frac{\mu_0 b^2 \omega I}{2a}$$

4. When the reading of G vanishes, $I_G = 0$ and Kirchoff laws give an immediate answer. Then we have

$$\mathcal{E} = IR \quad \Rightarrow \quad R = N\frac{\mu_0 b^2 \omega}{2a}$$

5. The force per unit length f between two indefinite parallel straight wires separated by a distance h is.

$$f = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{h}$$

for $I_1 = I_2 = I$ and length $2\pi a$, the force F induced on C_2 by the neighbor coils C_1 is

$$F = \frac{\mu_0 a}{h} I^2$$

6. In equilibrium

$$mgx = 4Fd$$

Then

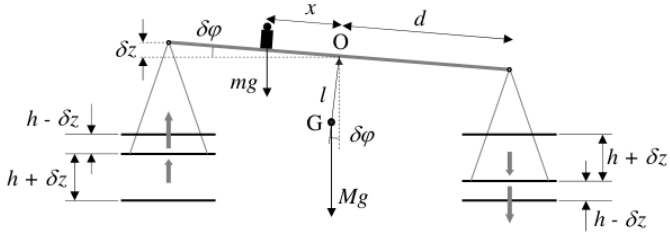
$$mgx = \frac{4\mu_0 ad}{h} I^2 \tag{1}$$

so that

$$I = \left(\frac{mghx}{4\mu_0 ad}\right)^{1/2}$$

7. The balance comes back towards the equilibrium position for a little angular deviation $\delta\varphi$ if the gravity torques with respect to the fulcrum O are greater than the magnetic torques.

$$Mgl\sin\delta\varphi + mgx\cos\delta\varphi > 2\mu_0 aI^2\left(\frac{1}{h-\delta z} + \frac{1}{h+\delta z}\right) d\cos\delta\varphi$$



Therefore, using the suggested approximation

$$Mgl\sin\delta\varphi + mgx\cos\delta\varphi > \frac{4\mu_0 adI^2}{h}\left(1 + \frac{\delta z^2}{h^2}\right)\cos\delta\varphi$$

Taking into account the equilibrium condition (1), one obtains

$$Mgl\sin\delta\varphi > mgx\frac{\delta z^2}{h^2}\cos\delta\varphi$$

Finally, for $\tan\delta\varphi \approx \sin\delta\varphi = \frac{\delta z}{d}$

$$\delta z < \frac{Mlh^2}{mxd} \quad \Rightarrow \quad \delta z_{\max} = \frac{Mlh^2}{mxd}$$

Th 2 ANSWER SHEET

Question	Basic formulas and ideas used	Analytical results	Marking guide
1	$\Phi = N\vec{B}_0 \cdot \vec{S}$ $\mathcal{E} = -\dfrac{d\Phi}{dt}$ $P = \dfrac{\mathcal{E}^2}{R}$	$\mathcal{E} = N\pi a^2 B_0 \omega \sin \omega t$ $\langle P \rangle = \dfrac{\left(N\pi a^2 B_0 \omega\right)^2}{2R}$	0.5 1.0
2	$\vec{B} = \vec{B}_0 + \vec{B}_i$ $B_i = \dfrac{\mu_0 N}{2a} I$ $\tan\theta = \dfrac{\langle B_y \rangle}{\langle B_x \rangle}$	$R = \dfrac{\mu_0 N^2 \pi a \omega}{4 \tan\theta}$	2.0
3	$\vec{E} = \vec{v} \times \vec{B}$ $v = \omega r$ $B = N\dfrac{\mu_0 I}{2a}$ $\mathcal{E} = \int_0^b \vec{E}\, d\vec{r}$	$\mathcal{E} = N\dfrac{\mu_0 b^2 \omega I}{2a}$	2.0
4	$\mathcal{E} = RI$	$R = N\dfrac{\mu_0 b^2 \omega}{2a}$	0,5
5	$f = \dfrac{\mu_0}{2\pi}\dfrac{I\, I'}{h}$	$F = \dfrac{\mu_0 a}{h} I^2$	1.0
6	$mgx = 4Fd$	$I = \left(\dfrac{mghx}{4\mu_0 ad}\right)^{1/2}$	1.0
7	$\Gamma_{grav} > \Gamma_{mag}$	$\delta z_{\max} = \dfrac{Mlh^2}{mxd}$	2.0