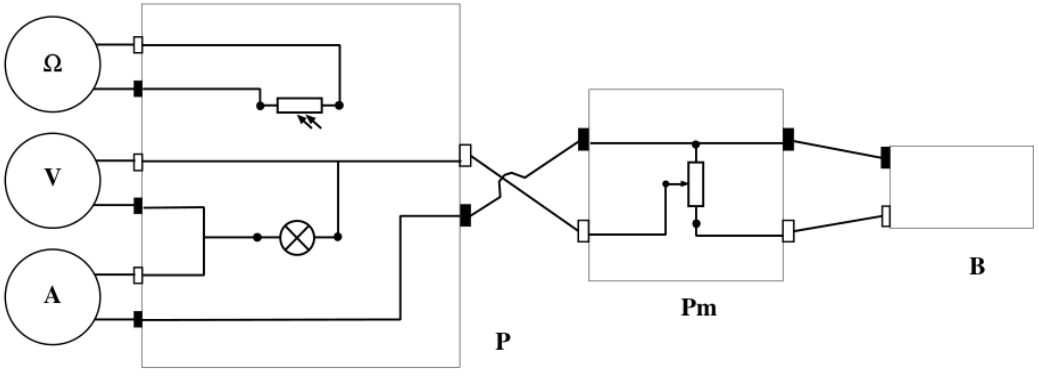
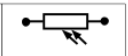
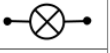
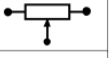
PLANCK'S CONSTANT IN THE LIGHT OF AN INCANDESCENT LAMP

SOLUTION

TASK 1

Draw the electric connections in the boxes and between boxes below.



Photoresistor	
Incandescent Bulb	
Potentiometer	
Red socket	
Black socket	

Ω Ohmmeter
V Voltmeter
A Ammeter
P Platform
Pm Potentiometer
B Battery

TASK 2

a)

$t_0 = 24\ ^{o}\mathrm{C}$ $T_0 = 297\ \mathrm{K}$ $\Delta T_0 = 1\ \mathrm{K}$

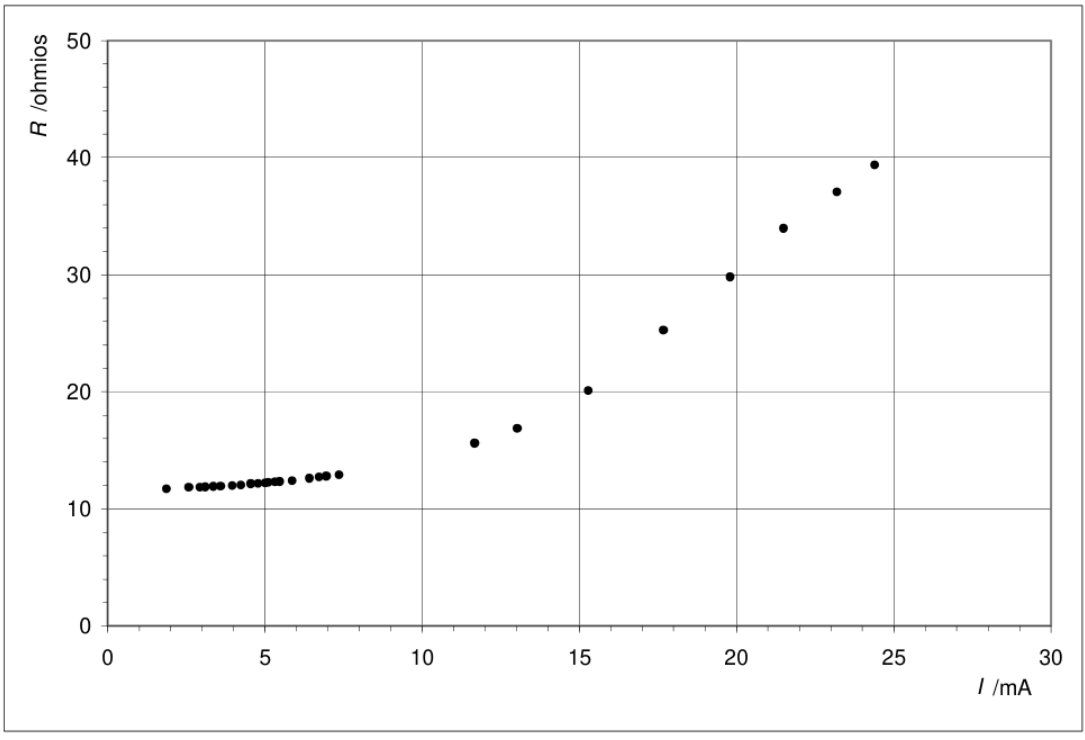
b)

V /mV	I / mA	R_B /Ω
21.9	1.87	11.7
30.5	2.58	11.8
34.9	2.95	11.8
37.0	3.12	11.9
40.1	3.37	11.9
43.0	3.60	11.9
47.6	3.97	12.0
51.1	4.24	12.1
55.3	4.56	12.1
58.3	4.79	12.2
61.3	5.02	12.2
65.5	5.33	12.3
67.5	5.47	12.3
73.0	5.88	12.4
80.9	6.42	12.6
85.6	6.73	12.7
89.0	6.96	12.8
95.1	7.36	12.9
111.9	8.38	13.4
130.2	9.37	13.9
181.8	11.67	15.6
220	13.04	16.9
307	15.29	20.1
447	17.68	25.1
590	19.8	29.8
730	21.5	33.9
860	23.2	37.1
960	24.4	39.3

$V_{min} = 9.2$ mV *

* This is a characteristic of your apparatus. You can´t go below it.

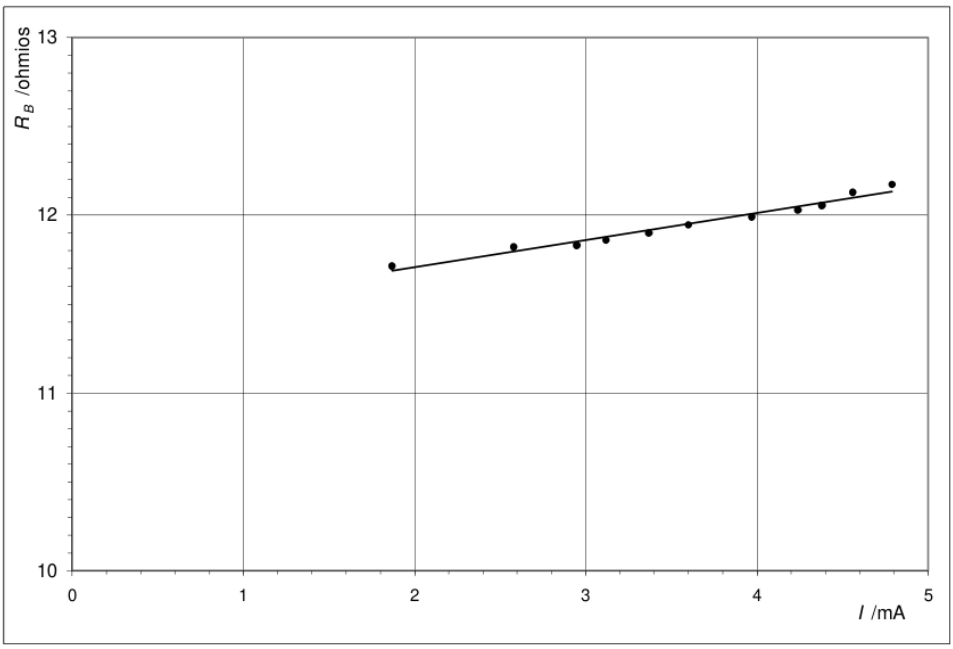
We represent R_B in the vertical axis against I.



In order to work out R_{B0} , we choose the first ten readings.

c)

V /mV	I / mA	R_B /Ω
21.9 ± 0.1	1.87 ± 0.01	11.7 ± 0.1
30.5 ± 0.1	2.58 ± 0.01	11.8 ± 0.1
34.9 ± 0.1	2.95 ± 0.01	11.8 ± 0.1
37.0 ± 0.1	3.12 ± 0.01	11.9 ± 0.1
40.1 ± 0.1	3.37 ± 0.01	11.9 ± 0.1
43.0 ± 0.1	3.60 ± 0.01	11.9 ± 0.1
47.6 ± 0.1	3.97 ± 0.01	12.0 ± 0.1
51.1 ± 0.1	4.24 ± 0.01	12.1 ± 0.1
55.3 ± 0.1	4.56 ± 0.01	12.1 ± 0.1
58.3 ± 0.1	4.79 ± 0.01	12.2 ± 0.1



Error for R_B (We work out the error for first value, as example).

$$\Delta R_B = R_B \sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta I}{I}\right)^2} = 11.71 \sqrt{\left(\frac{0.1}{21.9}\right)^2 + \left(\frac{0.01}{1.87}\right)^2} = 0.1$$

We have worked out R_{B0} by the least squares.

$R_{B0} = 11.4$

slope $= m = 0.167$

$\sum I^2 = 130.38$

$\sum I = 35.05$

$n = 10$

For axis X: $\sigma_I = \sqrt{\dfrac{\sum \Delta I^2}{n}} = 0.01$

For axis Y: $\sigma_{R_B} = \sqrt{\dfrac{\sum \Delta R_B^2}{n}} = 0.047$

$\sigma = \sqrt{\sigma_{R_B}^2 + m^2 \sigma_I^2} = \sqrt{0.1^2 + 0.167^2 \cdot 0.01^2} = 0.1$

$$\Delta R_{B0} = \sqrt{\frac{\sigma^2 \sum I^2}{n \sum I^2 - (\sum I)^2}} = \sqrt{\frac{0.1^2 \times 130.38}{10 \cdot 130.38 - 35.05^2}} = 0.13$$

$R_{B0} = 11{,}4\ \Omega$ $\Delta R_{B0} = 0.1\ \Omega$

d)

$$T = aR^{0.83}\ ; \quad a = \frac{T_0}{R_0^{0.83}}\ ; \quad a = \frac{297}{11.4^{0.83}} = 39.40$$

Working out the error for two methods:

Method A

$$\ln a = \ln T_0 - 0.83 \ln R_{B0}\ ;$$

$$\Delta a = a \left(\frac{\Delta T_0}{T_0} + 0.83 \frac{\Delta R_{B0}}{R_{B0}} \right)\ ;$$

$$\Delta a = 39.40 \left(\frac{1}{297} + 0.83 \frac{0.1}{11.40} \right) = 0.419 = 0.4$$

Method B

Higher value of a:

$$a_{\max} = \frac{T_0 + \Delta T_0}{(R_0 - \Delta R_0)^{0.83}} = \frac{297 + 1}{(11.4 - 0.1)^{0.83}} = 39.8255$$

Smaller value of a:

$$a_{\min} = \frac{T_0 - \Delta T_0}{(R_0 + \Delta R_0)^{0.83}} = \frac{297 - 1}{(11.4 + 0.1)^{0.83}} = 38.9863$$

$$\Delta a = \frac{a_{\max} - a_{\min}}{2} = \frac{39.8255 - 38.9863}{2} = 0.419 = 0.4$$

$a = 39.4$ $\Delta a = 0.4$

TASK 3

Because of

$$2\Delta\lambda = 620 - 565\ ; \quad \Delta\lambda = 28\ \text{nm}$$

$\lambda_0 = 590$ nm $\Delta\lambda = 28$ nm

TASK 4

a)

V /V	I / mA	R /kΩ
9.48	85.5	8.77
9.73	86.8	8.11
9.83	87.3	7.90
100.1	88.2	7.49
10.25	89.4	7.00
10.41	90.2	6.67
10.61	91.2	6.35
10.72	91.8	6.16
10.82	92.2	6.01
10.97	93.0	5.77
11.03	93.3	5.69
11.27	94.5	5.35
11.42	95.1	5.17
11.50	95.5	5.07

b)

Because of

$$\ln \frac{R}{R'} = \gamma \ln 0.512 \; ; \quad \gamma = \ln \frac{R}{R'} \Big/ \ln 0.51\ 2 = \ln \frac{5.07}{8.11} \Big/ \ln 0.512 = 0.702$$

For working out $\Delta\gamma$we know that:

$R \pm \Delta R = 5.07 \pm 0.01$ kΩ

$R' \pm \Delta R' = 8.11 \pm 0.01$ kΩ

Transmittance, $t = 51.2\,\%$

Working out the error for two methods:

Method A

$$\gamma = \frac{\ln R/R'}{\ln t} \; ; \quad \Delta\gamma = \frac{1}{\ln t}\left(\frac{\Delta R}{R} + \frac{\Delta R'}{R'}\right) = \frac{1}{\ln 0.512}\left(\frac{0.01}{5.07} + \frac{0.01}{8.11}\right) = 0.00479 \; ; \quad \Delta\gamma = 0$$

Method B

Higher value of γ:

$$\gamma_{\max} = \ln \frac{R - \Delta R}{R' + \Delta R'} \Big/ \ln \gamma = \ln \frac{5.07 - 0.01}{8.11 + 0.01} \Big/ \ln 0.512 = 0.70654$$

Smaller value of γ:

$$\gamma_{\max} = \ln \frac{R + \Delta R}{R' - \Delta R'} \Big/ \ln \gamma = \ln \frac{5.07 + 0.01}{8.11 - 0.01} \Big/ \ln 0.512 = 0.69696$$

$$\Delta\gamma = \frac{\gamma_{\max} - \gamma_{\min}}{2} = \frac{0.70654 - 0.69696}{2} = 0.00479 \; ; \quad \Delta\gamma = 0.005$$

$R = 5.07$ kΩ $\qquad$ $\gamma = 0.702$
$R' = 8.11$ kΩ $\qquad$ $\Delta\gamma = 0.005$

c)

We know that

$$R = c_3 e^{\frac{c_2 \gamma}{\lambda_0 T}} \tag{3}$$

then

$$\ln R = \ln c_3 + \frac{c_2\gamma}{\lambda_0 T}$$

Because of

$$T = aR_B^{0.83} \tag{6}$$

consequently

$$\ln R = \ln c_3 + \frac{c_2\gamma}{\lambda_0 a} R_B^{-0.83}$$

$$\ln R = \ln c_3 + \frac{c_2\gamma}{\lambda_0 a} R_B^{-0.83} \qquad \text{Eq. (9)}$$

d)

V /V	I / mA	R_B / Ω	T / K	$R_B^{-0.83}$ (S.I.)	R / kΩ	$\ln R$
9.48 ± 0.01	85.5 ± 0.1	110.9 ± 0.2	1962 ± 18	$(2.008 \pm 0.004)\,10^{-2}$	8.77 ± 0.01	2.171 ± 0.001
9.73 ± 0.01	86.8 ± 0.1	112.1 ± 0.2	1980 ± 18	$(1.990 \pm 0.004)\,10^{-2}$	8.11 ± 0.01	2.093 ± 0.001
9.83 ± 0.01	87.3 ± 0.1	112.6 ± 0.2	1987 ± 18	$(1.983 \pm 0.004)\,10^{-2}$	7.90 ± 0.01	2.067 ± 0.001
10.01 ± 0.01	88.2 ± 0.1	113.5 ± 0.2	2000 ± 18	$(1.970 \pm 0.004)\,10^{-2}$	7.49 ± 0.01	2.014 ± 0.001
10.25 ± 0.01	89.4 ± 0.1	114.7 ± 0.2	2018 ± 18	$(1.952 \pm 0.003)\,10^{-2}$	7.00 ± 0.01	1.946 ± 0.001
10.41 ± 0.01	90.2 ± 0.1	115.4 ± 0.2	2028 ± 18	$(1.943 \pm 0.003)\,10^{-2}$	6.67 ± 0.01	1.894 ± 0.002
10.61 ± 0.01	91.2 ± 0.1	116.3 ± 0.2	2041 ± 18	$(1.930 \pm 0.003)\,10^{-2}$	6.35 ± 0.01	1.849 ± 0.002
10.72 ± 0.01	91.8 ± 0.1	116.8 ± 0.2	2049 ± 19	$(1.923 \pm 0.003)\,10^{-2}$	6.16 ± 0.01	1.818 ± 0.002
10.82 ± 0.01	92.2 ± 0.1	117.4 ± 0.2	2057 ± 19	$(1.915 \pm 0.003)\,10^{-2}$	6.01 ± 0.01	1.793 ± 0.002
10.97 ± 0.01	93.0 ± 0.1	118.0 ± 0.2	2066 ± 19	$(1.907 \pm 0.003)\,10^{-2}$	5.77 ± 0.01	1.753 ± 0.002
11.03 ± 0.01	93.3 ± 0.1	118.2 ± 0.2	2069 ± 19	$(1.904 \pm 0.003)\,10^{-2}$	5.69 ± 0.01	1.739 ± 0.002
11.27 ± 0.01	94.5 ± 0.1	119.3 ± 0.2	2085 ± 19	$(1.890 \pm 0.003)\,10^{-2}$	5.35 ± 0.01	1.677 ± 0.002
11.42 ± 0.01	95.1 ± 0.1	120.1 ± 0.2	2096 ± 19	$(1.880 \pm 0.003)\,10^{-2}$	5.15 ± 0.01	1.639 ± 0.002
11.50 ± 0.01	95.5 ± 0.1	120.4 ± 0.2	2101 ± 19	$(1.875 \pm 0.003)\,10^{-2}$	5.07 ± 0.01	1.623 ± 0.002
			unnecessary			

We work out the errors for all the first row, as example.

Error for R_B:

$$\Delta R_B = R_B \sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta I}{I}\right)^2} = 110.9 \sqrt{\left(\frac{0.01}{9.48}\right)^2 + \left(\frac{0.1}{85.5}\right)^2} = 0.2\ \Omega$$

Error for T:

$$\Delta T = T\left(\frac{\Delta a}{a} + 0.83\,\frac{\Delta R_B}{R_B}\right); \quad \Delta T = 1962\left(\frac{0.3}{39.4} + 0.83\,\frac{0.2}{110.9}\right) = 18\ \text{K}$$

Error for $R_B^{-0.83}$:

$$x = R_B^{-0.83}; \quad \ln x = -0.83 \ln R_B; \quad \Delta x = x \cdot 0.83\,\frac{\Delta R_B}{R_B}; \quad \Delta\left(R_B^{-0.83}\right) = R_B^{-0.83}\,\frac{\Delta R_B}{R_B}$$

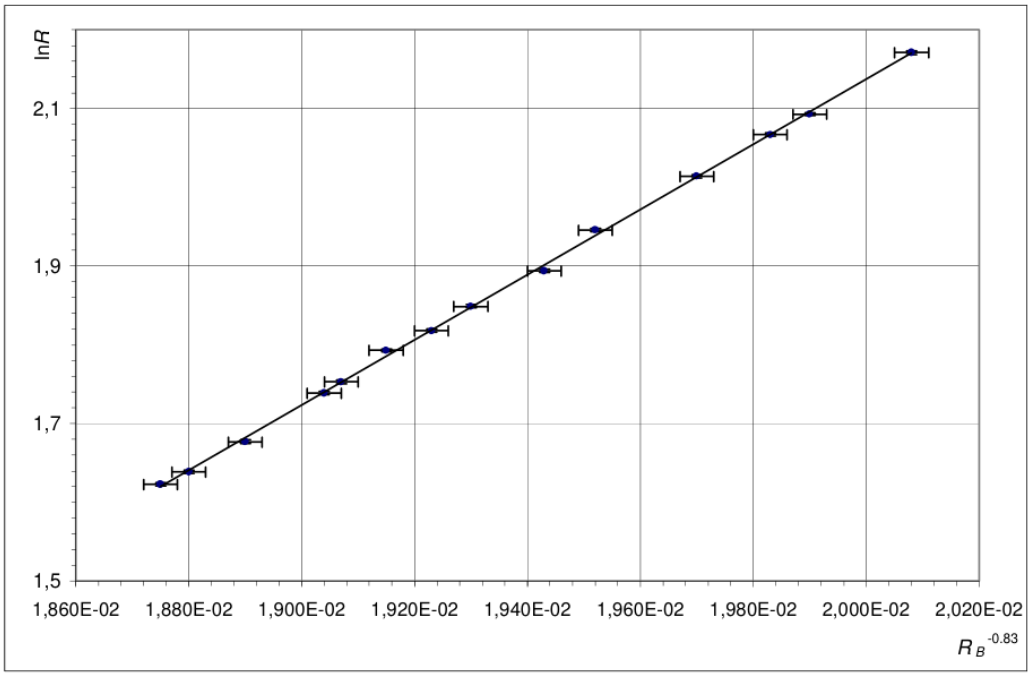
$$\Delta\left(R_B^{-0.83}\right) = 0.020077\,\frac{0.2}{110.9} \approx 0.004 \times 10^{-2}$$

Error for $\ln R$:

$$\Delta \ln R = \frac{\Delta R}{R}; \quad \Delta \ln R = \frac{0.01}{8.77} = 0.001$$

e)

We plot $\ln R$ versus $R_B^{-0.83}$.



By the least squares

Slope $= m = 414{,}6717$

$$\sum \left(R_B^{-0.83}\right)^2 = 5.23559 \times 10^{-3}$$

$$\sum \left(R_B^{-0.83}\right) = 0.27068$$

$n = 14$

For axis X: $\sigma_{R_B^{-0.83}} = \sqrt{\dfrac{\sum \Delta \left(R_B^{-0.83}\right)^2}{n}} = 0.003 \times 10^{-2}$

For axis Y: $\sigma_{\ln R} = \sqrt{\dfrac{\sum \Delta \left(\ln R\right)^2}{n}} = 0.002$

$$\sigma = \sqrt{\sigma_{\ln R}^2 + m^2 \sigma_{R_B^{-0.83}}^2} = \sqrt{0.002^2 + 414.672^2 \cdot \left(0.003 \times 10^{-2}\right)^2} = 0.0126$$

$$\Delta m = \sqrt{\frac{n\sigma^2}{n \sum \left(R_B^{-0.83}\right)^2 - \left(\sum R_B^{-0.83}\right)^2}} = \sqrt{\frac{14 \cdot 0{,}0126^2}{14 \cdot 5.23559 \times 10^{-3} - (0.27068)^2}} = 8.295$$

Because of

$$m = \frac{c_2 \gamma}{\lambda_0 a}$$

and

$$c_2 = \frac{hc}{k}$$

then

$$h = \frac{mk\lambda_0 a}{c\gamma}$$

$$h = \frac{414.67 \cdot 1.381 \times 10^{-23} \cdot 590 \times 10^{-9} \cdot 39.4}{2.998 \times 10^{8} \cdot 0.702} = 6.33 \times 10^{-34}$$

$$\Delta h = h\sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta k}{k}\right)^2 + \left(\frac{\Delta \lambda_0}{\lambda_0}\right)^2 + \left(\frac{\Delta a}{a}\right)^2 + \left(\frac{\Delta \gamma}{\gamma}\right)^2}$$

$$\Delta h = 6.34 \times 10^{-34}\sqrt{\left(\frac{8.3}{415}\right)^2 + 0 + \left(\frac{28}{590}\right)^2 + \left(\frac{0.3}{39.4}\right)^2 + 0 + \left(\frac{0.01}{0.70}\right)^2} = 0.34 \times 10^{-34}$$

$h = 6.3 \times 10^{-34}$ J · s $\qquad\qquad$ $\Delta h = 0.3 \times 10^{-34}$ J · s