Solution

Task 1

1.

1.1. $T_0 = 25 \pm 1\,^{\circ}\mathrm{C}$

$$V_{\mathrm{samp}}(T_0) = 573.9 \text{ mV}$$

With different experiment sets, V_{samp} may differ from the above value within ± 40 mV.

Note for error estimation:

δV and δV are calculated using the specs of the multimeter: $\pm 0.5\%$ reading digit $+2$ on the last digit. Example: if $V = 500\,\mathrm{mV}$, the error $\delta V = 500 \times 0.5\% + 0.2 = 2.7\,\mathrm{mV} \approx 3\,\mathrm{mV}$.

Thus, $V_{samp}(T_0) = 574 \pm 3$ mV.

All values of $V_{samp}(T_0)$ within $505 \div 585$ mV are acceptable.

1.2. Formula for temperature calculation:

From Eq (1): $V_{\mathrm{samp}} = V_{\mathrm{samp}}(T_0) - \alpha(T - T_0)$

$$V_{\mathrm{samp}}\,(50\,^{\circ}\mathrm{C}) = 523.9 \text{ mV}$$

$$V_{\mathrm{samp}}\,(70\,^{\circ}\mathrm{C}) = 483.9 \text{ mV}$$

$$V_{\mathrm{samp}}\,(80\,^{\circ}\mathrm{C}) = 463.9 \text{ mV}$$

Error calculation: $\delta V_{\mathrm{samp}} = \delta V_{\mathrm{samp}}(T_0) + (T - T_0)\,\delta\alpha$

Example: $V_{\mathrm{samp}} = 495.2$ mV, then $\delta V_{\mathrm{samp}} = 2.7 + 0.03 \times (50 - 25) = 3.45$ mV ≈ 3.5 mV

Thus:

$$V_{\mathrm{samp}}\,(50\,^{\circ}\mathrm{C}) = 524 \pm 4 \text{ mV}$$

$$V_{\mathrm{samp}}\,(70\,^{\circ}\mathrm{C}) = 484 \pm 4 \text{ mV}$$

$$V_{\mathrm{samp}}\,(80\,^{\circ}\mathrm{C}) = 464 \pm 5 \text{ mV}$$

The same rule for acceptable range of V_{samp} as in 1.1 is applied.

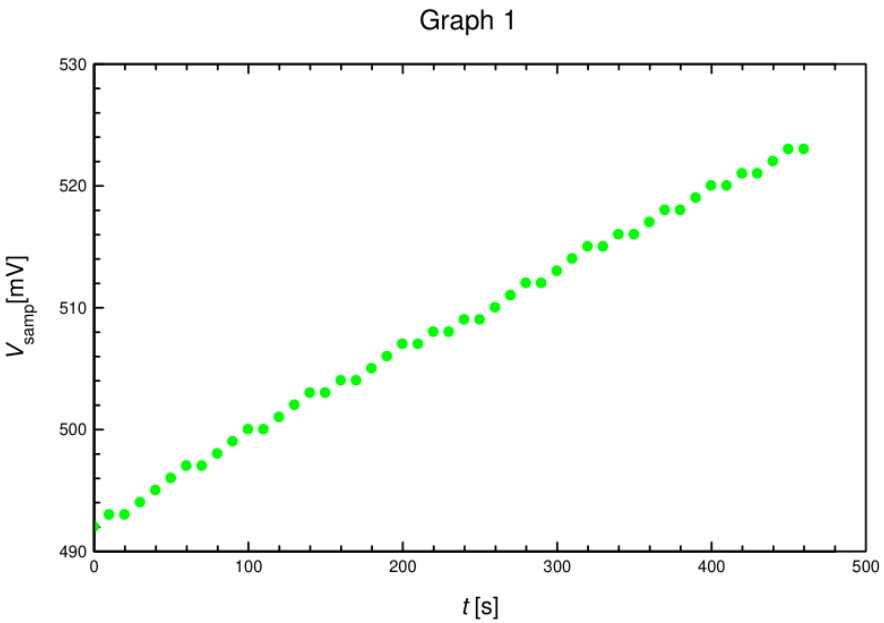
2.

2.1. Data of cooling-down process without sample:

t (s)	V_{samp} (mV) ($\pm$3mV)	ΔV (mV) ($\pm$0.2mV)
0	492	-0.4
10	493	-0.5
20	493	-0.5
30	494	-0.6
40	495	-0.7
50	496	-0.7
60	497	-0.8
70	497	-0.8
80	498	-0.9
90	499	-1.0
100	500	-1.0
110	500	-1.1
120	501	-1.1
130	502	-1.2
140	503	-1.2
150	503	-1.3
160	504	-1.3
170	504	-1.4
180	505	-1.5
190	506	-1.6
200	507	-1.6
210	507	-1.7
220	508	-1.7
230	508	-1.8
240	509	-1.8
250	509	-1.8
260	510	-1.9
270	511	-1.9
280	512	-1.9
290	512	-2.0
300	513	-2.0
310	514	-2.1
320	515	-2.1
330	515	-2.1
340	516	-2.1
350	516	-2.2
360	517	-2.2
370	518	-2.3
380	518	-2.3
390	519	-2.3
400	520	-2.4
410	520	-2.4
420	521	-2.5
430	521	-2.5
440	522	-2.5
450	523	-2.6
460	523	-2.6

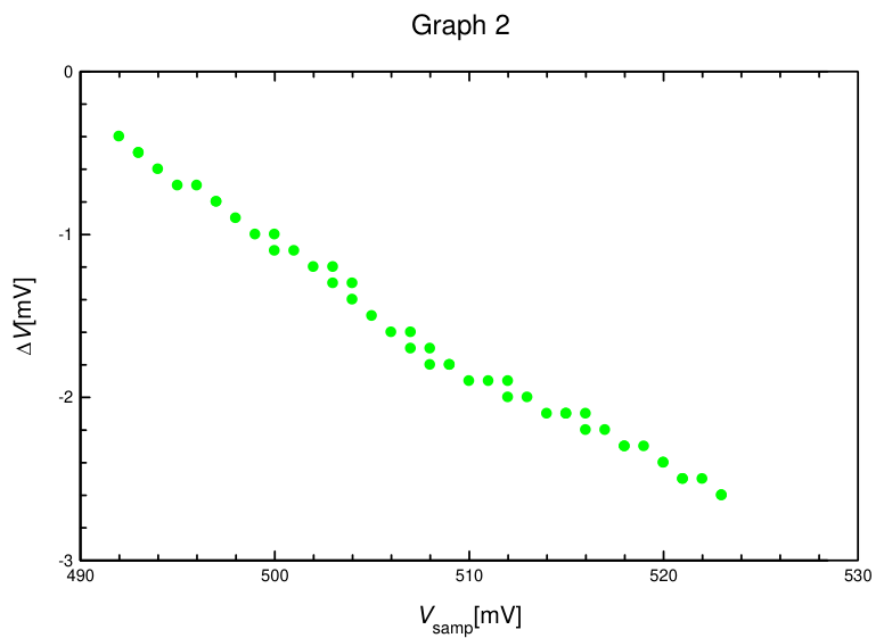
The acceptable range of ΔV is $\pm$40 mV. There is no fixed rule for the change in ΔV with T (this depends on the positions of the dishes on the plate, etc.)

2.2.



The correct graph should not have any abrupt changes of the slope.

2.3.



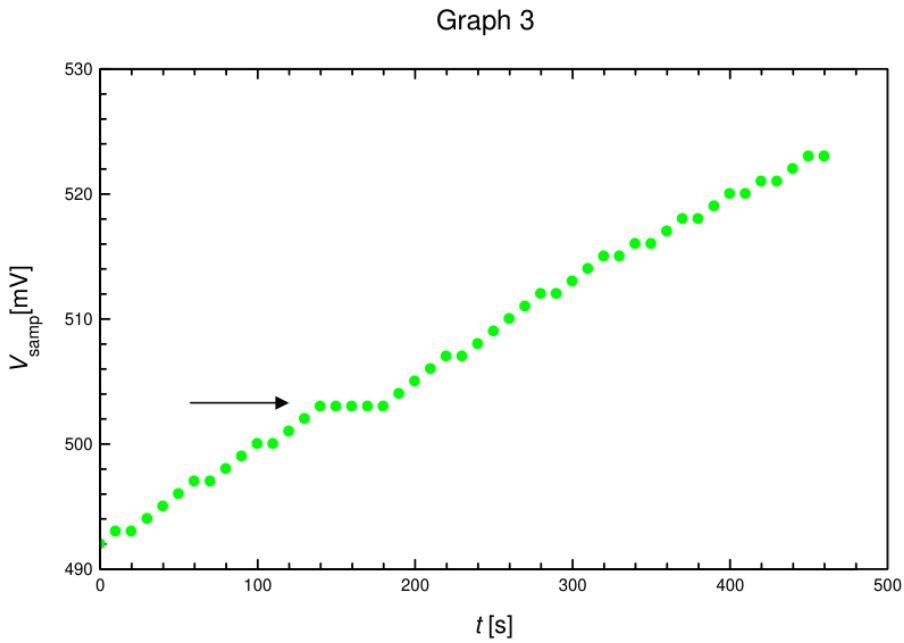
The correct graph should not have any abrupt changes of the slope.

3.

3.1. Dish with substance

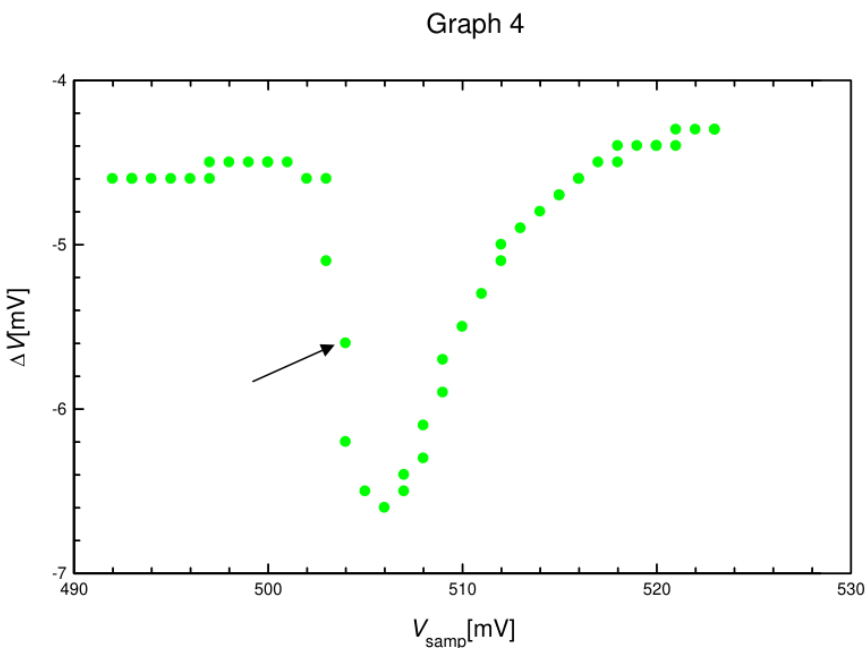
t (s)	V_{samp} (mV) ($\pm$3mV)	ΔV (mV) ($\pm$0.2mV)
0	492	-4.6
10	493	-4.6
20	493	-4.6
30	494	-4.6
40	495	-4.6
50	496	-4.6
60	497	-4.6
70	497	-4.5
80	498	-4.5
90	499	-4.5
100	500	-4.5
110	500	-4.5
120	501	-4.5
130	502	-4.6
140	503	-4.6
150	503	-5.1
160	503	-5.6
170	503	-6.2
180	503	-6.5
190	504	-6.6
200	505	-6.5
210	506	-6.4
220	507	-6.3
230	507	-6.1
240	508	-5.9
250	509	-5.7
260	510	-5.5
270	511	-5.3
280	512	-5.1
290	512	-5.0
300	513	-4.9
310	514	-4.8
320	515	-4.7
330	515	-4.7
340	516	-4.6
350	516	-4.6
360	517	-4.5
370	518	-4.5
380	518	-4.4
390	519	-4.4
400	520	-4.4
410	520	-4.4
420	521	-4.4
430	521	-4.3
440	522	-4.3
450	523	-4.3
460	523	-4.3

3.2.



The correct Graph 3 should contain a short plateau as marked by the arrow in the above figure.

3.3.



The correct Graph 4 should have an abrupt change in ΔV, as shown by the arrow in the above figure.

Note: when the dish contains the substance, values of ΔV may change compared to those without the substance.

4.

4.1. V_s is shown in Graph 3. Value $V_s = (503 \pm 3)$ mV. From that, $T_s = 60.5\,^{\circ}\mathrm{C}$ can be deduced.

4.2. V_s is shown in Graph 4. Value $V_s = (503 \pm 3)$ mV. From that, $T_s = 60.5\,^{\circ}\mathrm{C}$ can be deduced.

4.3. Error calculations, using root mean square method:

Error of T_s: $T_s = T_0 + \dfrac{V(T_0) - V(T_s)}{\alpha} = T_0 + A$, in which A is an intermediate variable.

Therefore error of T_s can be written as $\delta T_s = \sqrt{(\delta T_0)^2 + (\delta A)^2}$, in which $\delta\ldots$ is the error.

Error for A is calculated separately:

$$\delta A = \frac{V(T_0) - V(T_s)}{\alpha}\sqrt{\left\{\frac{\delta[V(T_0) - V(T_s)]}{V(T_0) - V(T_s)}\right\}^2 + \left(\frac{\delta\alpha}{\alpha}\right)^2}$$

in which we have:

$$\delta[V(T_0) - V(T_s)] = \sqrt{[\delta V(T_0)]^2 + [\delta V(T_s)]^2}$$

Errors of other variables in this experiment:

$$\delta T_0 = 1\,^{\circ}\mathrm{C}$$

$\delta V(T_0) = 3$ mV, read on the multimeter.

$$\delta\alpha = 0.03\ \mathrm{mV}/^{\circ}\mathrm{C}$$

$$\delta V(T_\mathrm{s}) \approx 3\ \mathrm{mV}$$

From the above constituent errors we have:

$$\delta[V(T_0) - V(T_\mathrm{s})] \approx 4.24\ \mathrm{mV}$$

$$\delta A \approx 2.1\,^{\circ}\mathrm{C}$$

Finally, the error of T_s is: $\delta T_\mathrm{s} \approx 2.5\,^{\circ}\mathrm{C}$

Hence, the final result is: $T_\mathrm{s} = 60 \pm 2.5\,^{\circ}\mathrm{C}$

Note: if the student uses any other reasonable error calculation method that leads to approximately the same result, it is also accepted.

Task 2

1.

1.1. $T_0 = 26 \pm 1\,^{\circ}\mathrm{C}$

2.

2.1. Measured data with the lamp off

t (s)	$\Delta V(T_0)$ (mV) ($\pm$0.2mV)
0	19.0
10	19.0
20	19.0
30	19.0
40	19.0
50	18.9
60	18.9
70	18.9
80	18.9
90	18.9
100	19.0
110	19.0
120	19.0

Values of $\Delta V(T_0)$ can be different from one experiment set to another. The acceptable values lie in between -40÷+40 mV.

2.2. Measured data with the lamp on

t (s)	ΔV (mV) ($\pm$0.2mV)
0	19.5
10	21.9
20	23.8
30	25.5
40	26.9
50	28.0
60	29.0
70	29.9
80	30.7
90	31.4
100	32.0
110	32.4
120	32.9

When illuminated (by the lamp) values of ΔV may change $10 \div 20$ mV compared to the initial situation (lamp off).

2.3. Measured data after turning the lamp off

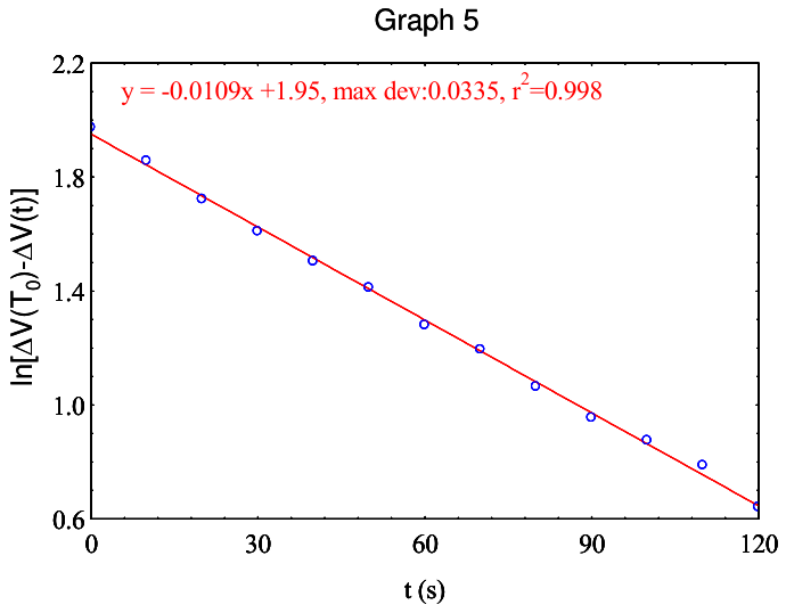
t (s)	ΔV (mV) ($\pm$0.2mV)
0	23.2
10	22.4
20	21.6
30	21.0
40	20.5
50	20.1
60	19.6
70	19.3
80	18.9
90	18.6
100	18.4
110	18.2
120	17.9

3. Plotting graph 5 and calculating k

3.1. $x = t$; $y = \mathbf{ln}\,[\Delta V(T_0) - \Delta V(t)]$

Note: other reasonable ways of writing expressions for x and y that also leads to a linear relationship using **ln** are also accepted.

3.2. Graph 5



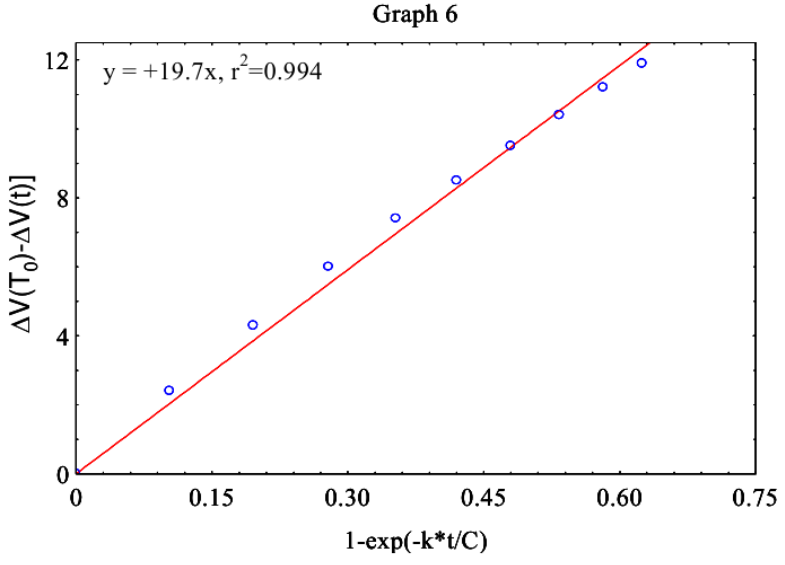
3.3. Calculating k: $\dfrac{k}{C} = 0.0109\ \text{s}^{-1}$ and $C = 0.69$ J/K, thus: $k = 7.52 \times 10^{-3}$ W/K

Note: Error of k will be calculated in 5.5. Students are not asked to give error of k in this step. The acceptable value of k lies in between $6 \times 10^{-3} \div 9 \times 10^{-3}$ W/K depending on the experiment set.

4. Plotting Graph 6 and calculating E

4.1. $x = \left[1 - \exp\left(\dfrac{-kt}{C}\right)\right]$; $y = |\Delta V(T_0) - \Delta V(t)|$

4.2.



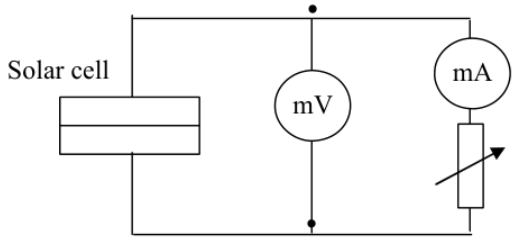
Graph 6 should be substantially linear, with the slope in between 15÷25 mV, depending on the experiment set.

4.3. From the slope of Graph 6 and the area of the detector orifice we obtain $E = 140$ W/m^2. The area of the detector orifice is $S_{\text{det}} = \pi R_{\text{det}}^2 = \pi \times (13 \times 10^{-3})^2 = 5.30 \times 10^{-4}$ m^2 with error: $\dfrac{\delta R_{\text{det}}}{R_{\text{det}}} = 5\%$

Error of E will be calculated in 5.5. Students are not asked to give error of E in this step. The acceptable value of E lies in between $120 \div 160$ W/m^2, depending on the experiment set.

5.

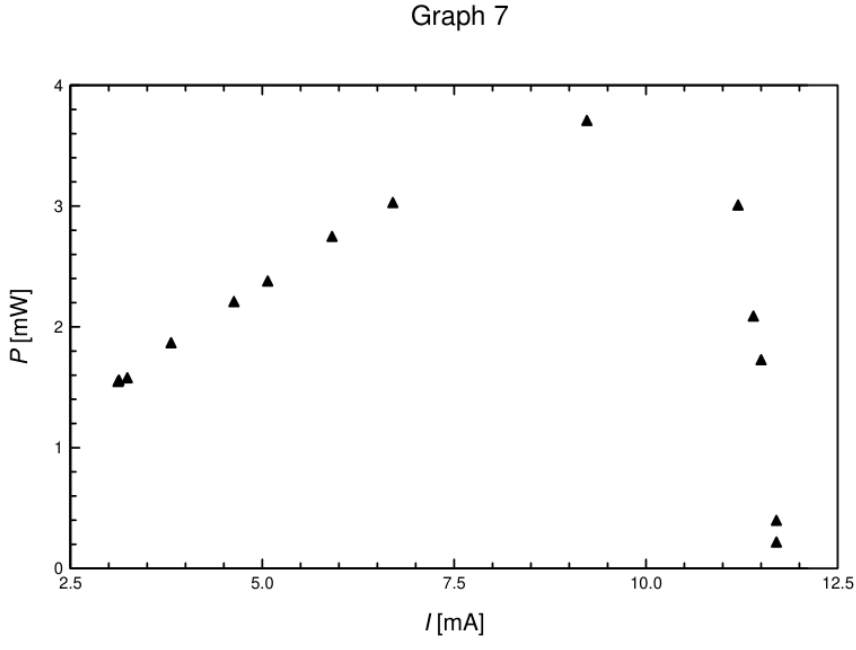
5.1. Circuit diagram:



5.2. Measurements of V and I

V (mV) ($\pm 0.3 \div 3$ mV)	I (mA) ($\pm 0.05 \div 0.1$ mA)	P (mW)
18.6 ± 0.3	11.7	0.21
33.5	11.7	0.39
150	11.5	1.72
157	11.6	1.82
182 ± 1	11.4	2.08
267	11.2	3.00
402 ± 2	9.23	3.70
448	6.70	3.02
459	5.91	2.74
468	5.07	2.37
473 ± 3	4.63	2.20
480	3.81	1.86
485	3.24	1.57
487	3.12	1.54
489	3.13	1.55

5.3.



5.4. $P_{\max} = 3.7 \pm 0.2$ mW

The acceptable value of $P_{\max}$ lies in between $3 \div 4.5$ mW, depending on the experiment set.

5.5. Expression for the efficiency

$$S_{\text{cell}} = 19 \times 24 \text{ mm}^2 = 450 \times 10^{-6} \text{ m}^2$$

Then $\eta_{\max} = \dfrac{P_{\max}}{E \times S_{\text{cell}}} = 0.058$

Error calculation:

$$\delta\eta_{\max} = \eta_{\max}\sqrt{\left(\frac{\delta P_{\max}}{P_{\max}}\right)^2 + \left(\frac{\delta E}{E}\right)^2 + \left(\frac{\delta S_{\text{cell}}}{S_{\text{cell}}}\right)^2}$$

, in which S_{cell} is the area of the solar cell.

$\dfrac{\delta P_{\max}}{P_{\max}}$ is estimated from Graph 7, typical value $\approx 6\,\%$

$\dfrac{\delta S_{\text{cell}}}{S_{\text{cell}}}$: error from the millimeter measurement (with the ruler), typical value $\approx 5\,\%$

E is calculated from averaging the ratio (using Graph 6):

$$B = \frac{\Delta V(T_0) - \Delta V(t)}{1 - \mathbf{exp}\left(-\frac{k}{C}t\right)} = \frac{E\pi R_{\text{det}}^2\alpha}{k}$$

in which B is an intermediate variable, R_{det} is the radius of the detector orifice.

$$E = \frac{kB}{\pi R_{\text{det}}^2\alpha}$$

Calculation of error of E:

$$\overline{\left(\frac{\delta E}{E}\right)} = \sqrt{\left(\frac{\delta k}{k}\right)^2 + \left(\frac{\delta B}{B}\right)^2 + 4\left(\frac{\delta R_{\text{det}}}{R_{\text{det}}}\right)^2 + \left(\frac{\delta\alpha}{\alpha}\right)^2}$$

k is calculated from the regression of:

$$\Delta T = \Delta T(0)\mathbf{exp}\left(-\frac{k}{C}t\right), \text{hence } \mathbf{ln}\,\Delta T = \mathbf{ln}\,\Delta T(0) - \frac{k}{C}t$$

We set $k/C = m$ then $k = mC$

From the regression, we can calculate the error of m:

$$\frac{\delta m}{m} \approx 2(1 - r) \approx 0.2\%$$

$$\frac{\delta k}{k} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta C}{C}\right)^2}$$

We derive the expression for the error of $\eta_{\max}$:

$$\delta\eta_{\max} = \eta_{\max}\sqrt{\left(\frac{\delta P_{\max}}{P_{\max}}\right)^2 + \left(\frac{\delta S_{\text{cell}}}{S_{\text{cell}}}\right)^2 + \left(\frac{\delta B}{B}\right)^2 + 4\left(\frac{\delta R_{\text{det}}}{R_{\text{det}}}\right)^2 + \left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta C}{C}\right)^2 + \left(\right.}$$

Typical values for $\eta_{\max}$ and other constituent errors:

$$\eta_{\max} \approx 0.058$$

$$\frac{\delta P_{\max}}{P_{\max}} = 5\%\,; \qquad \frac{\delta B}{B} \approx 0.6\%\,; \qquad \frac{\delta m}{m} \approx 0.2\%\,; \qquad \frac{\delta S_{\text{cell}}}{S_{\text{cell}}} \approx 5\%\,; \qquad \frac{\delta R_{\text{det}}}{R_{\text{det}}} \approx 5\%\,;$$

$$\frac{\delta C}{C} \approx 3\% \; ; \qquad \frac{\delta k}{k} \approx 3\% \; ; \qquad \frac{\delta E}{E} \approx 10.5\% \; ; \qquad \frac{\delta \alpha}{\alpha} \approx 1.5\%$$

Finally:

$$\frac{\delta \eta_{\max}}{\eta_{\max}} = 12.7\% \; ; \qquad \delta \eta_{\max} \approx 0.0074$$

and

$$\eta_{\max} = (5.8 \pm 0.8)\,\%$$

Note: if the student uses any other reasonable error method that leads to approximately the same result, it is also accepted.