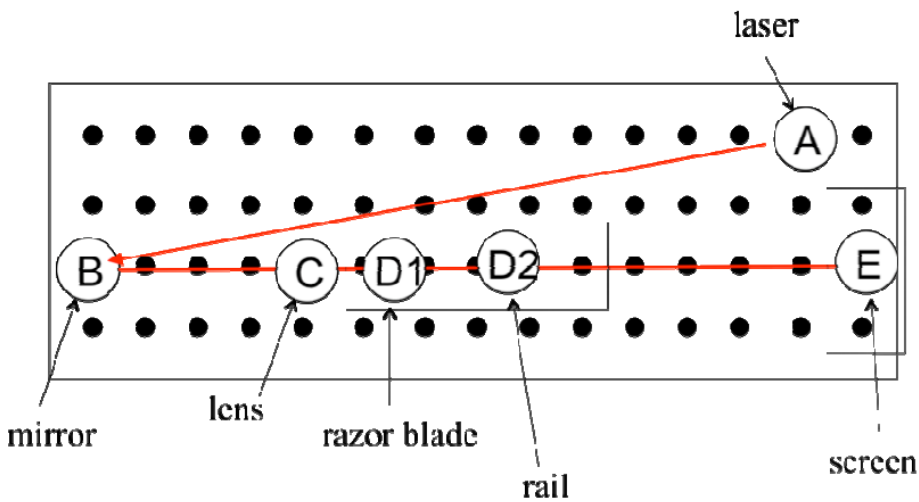
Answer Form

Experimental Problem No. 1

Diode laser wavelength

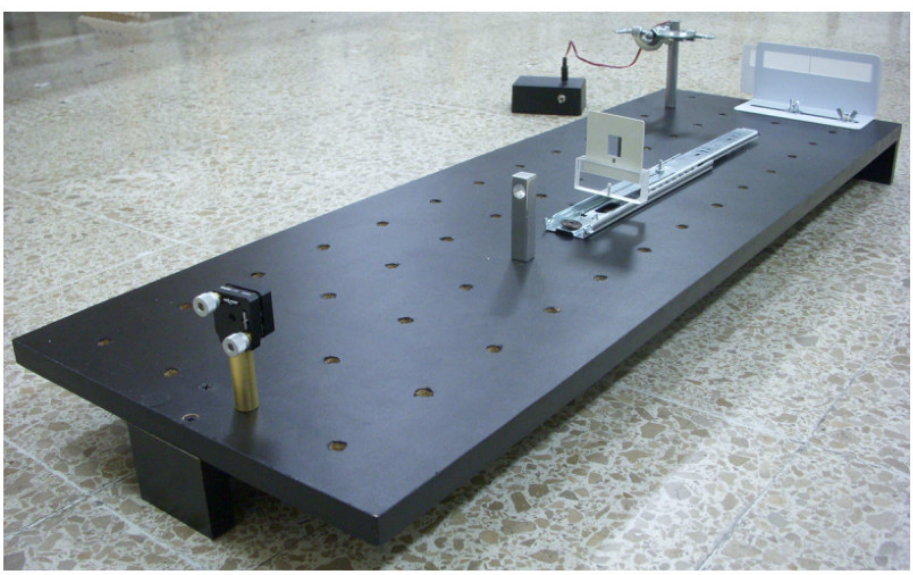
Task 1.1 Experimental setup.

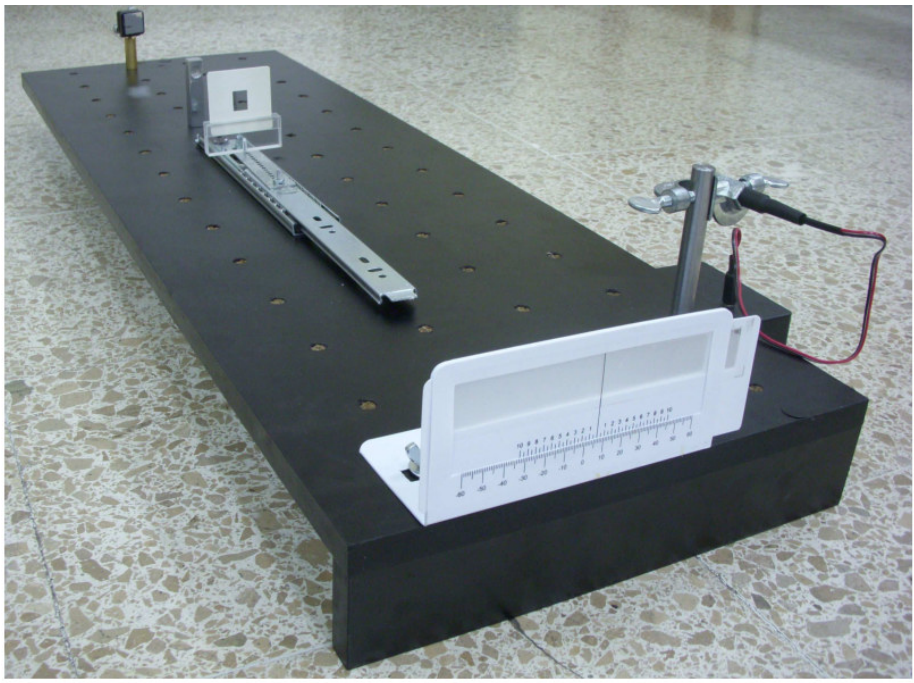


(0.75)

1.1 Sketch the laser path in drawing of Task 1.1 and Write down the height h of the beam as measured from the table 1.0

$h \pm \Delta h = (5.0 \pm 0.05) \times 10^{-2}\,\mathrm{m}$ (0.25)





Experimental setup for measurement of diode laser wavelength

Task 1.2 Expressions for optical path differences.

1.2 The path differences are 0.5

Case I: (0.25)

$$\Delta_{\mathrm{I}}(n) = (BF + FP) - BP = (L_b - L_0) + \sqrt{L_0^2 + L_R^2(n)} - \sqrt{L_b^2 + L_R^2(n)}$$

$$= (L_b - L_0) + L_0\sqrt{1 + \frac{L_R^2(n)}{L_0^2}} - L_b\sqrt{1 + \frac{L_R^2(n)}{L_b^2}}$$

using $\sqrt{1+x} \approx 1 + \frac{1}{2}x$

$$\approx (L_b - L_0) + L_0\left(1 + \frac{1}{2}\frac{L_R^2(n)}{L_0^2}\right) - L_b\left(1 + \frac{1}{2}\frac{L_R^2(n)}{L_b^2}\right)$$

$$\Rightarrow \quad \Delta_{\mathrm{I}}(n) \approx \frac{1}{2}L_R^2(n)\left(\frac{1}{L_0} - \frac{1}{L_b}\right)$$

Case II: (0.25)

$$\Delta_{\mathrm{II}}(n) = (FB + BP) - FP = (L_0 - L_a) + \sqrt{L_a^2 + L_L^2(n)} - \sqrt{L_0^2 + L_L^2(n)}$$

$$\approx (L_0 - L_a) + L_a\sqrt{1 + \frac{L_L^2(n)}{L_a^2}} - L_0\sqrt{1 + \frac{L_L^2(n)}{L_0^2}}$$

using $\sqrt{1+x} \approx 1 + \frac{1}{2}x$

$$\approx (L_0 - L_a) + L_a\left(1 + \frac{1}{2}\frac{L_L^2(n)}{L_a^2}\right) - L_0\left(1 + \frac{1}{2}\frac{L_L^2(n)}{L_0^2}\right)$$

$$\Rightarrow \quad \Delta_{\mathrm{II}}(n) \approx \frac{1}{2}L_L^2(n)\left(\frac{1}{L_a} - \frac{1}{L_0}\right)$$

Task 1.3 Measuring the dark fringe positions and locations of the blade. Use additional sheets if necessary.

TABLE I

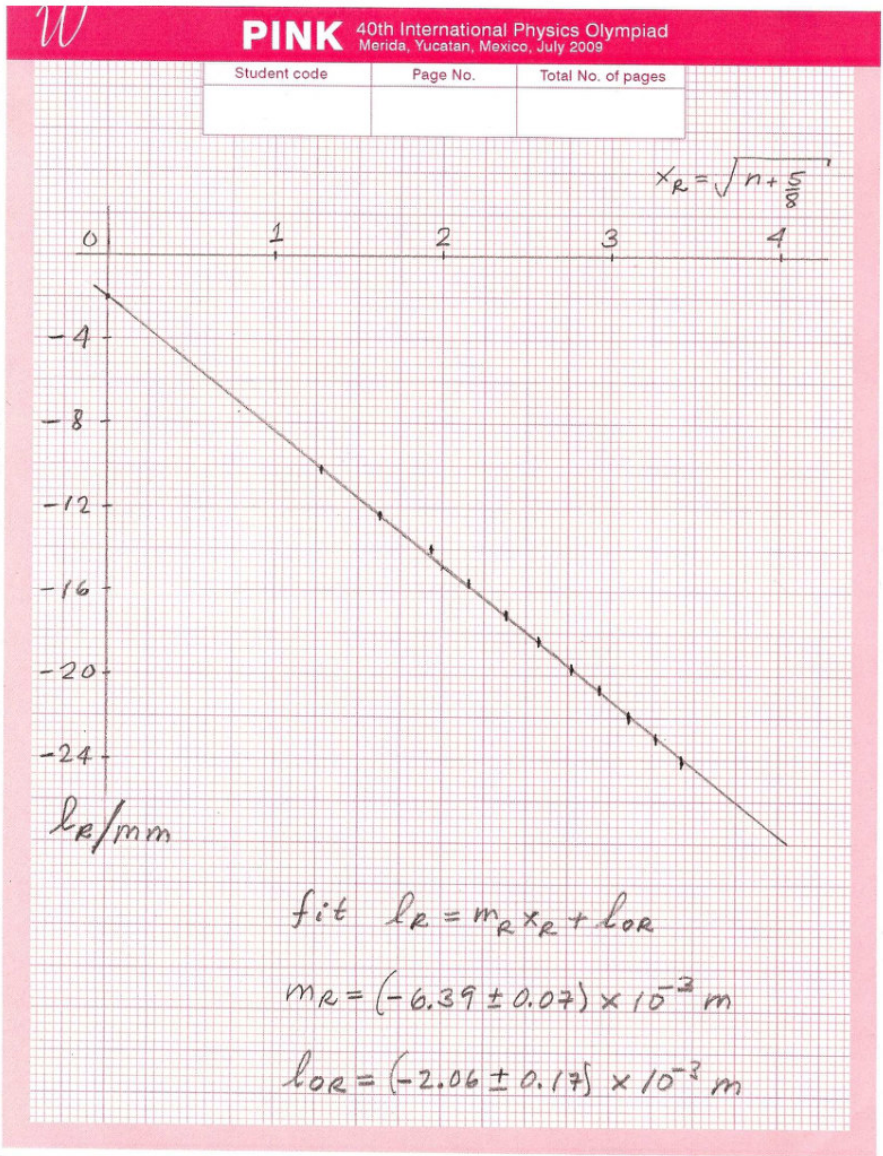
n	$(l_{\mathrm{R}}(n) \pm 0.1) \times 10^{-3}\,\mathrm{m}$	$(l_L(n) \pm 0.1) \times 10^{-3}\,\mathrm{m}$	x_R	x_L
0	-7.5	1.1	0.791	0.935
1	-10.1	3.7	1.275	1.369
2	-12.4	6.4	1.620	1.696
3	-14.0	8.2	1.903	1.968
4	-15.6	10.0	2.151	2.208
5	-17.2	11.4	2.372	2.424
6	-18.4	12.2	2.574	2.622
7	-19.7		2.761	
8	-20.7		2.937	
9	-22.0		3.102	
10	-23.0		3.260	
11	-24.1		3.410	

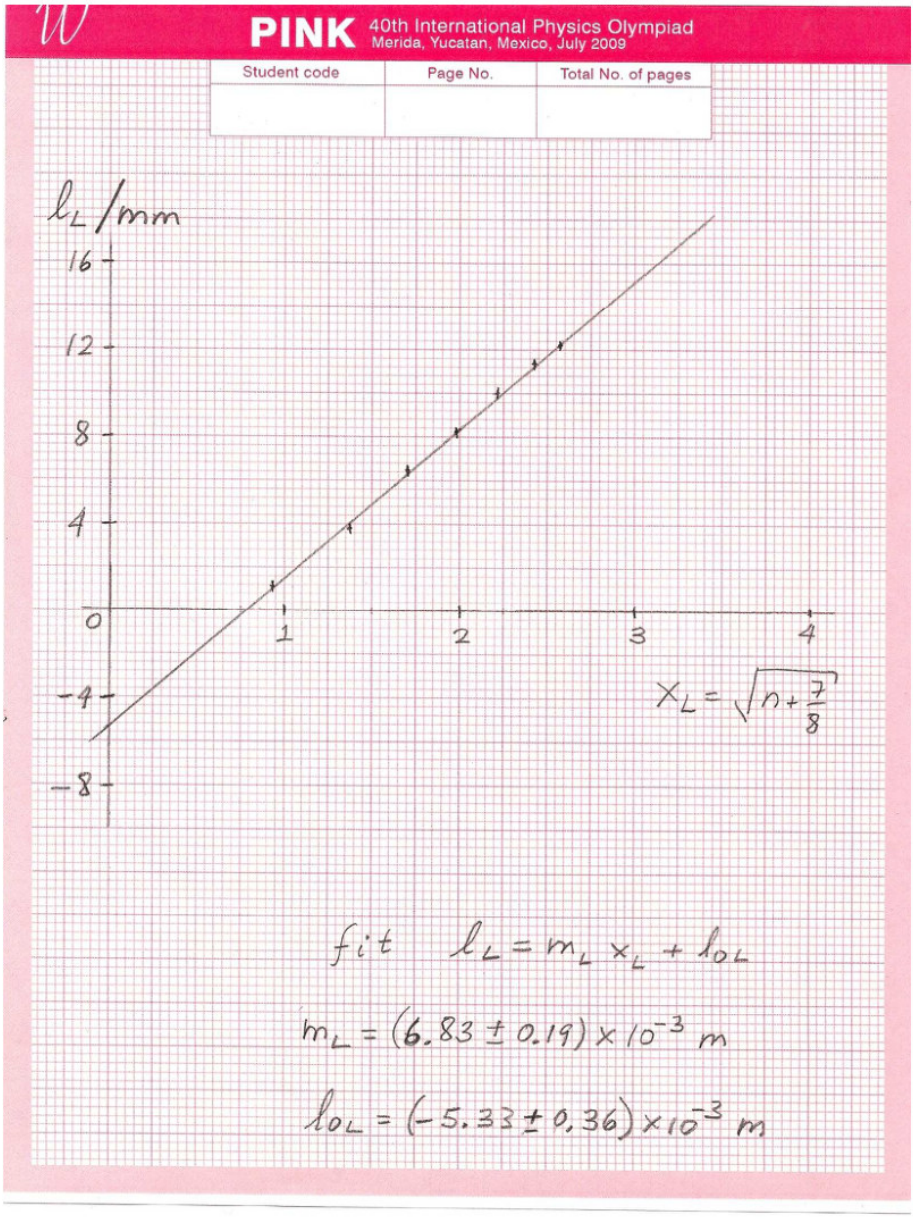
1.3 Report positions of the blade and their difference with higher precision: 3.25

$L_b \pm \Delta L_b = (653 \pm 1) \times 10^{-3}\,\mathrm{m}$ (0.25) LABEL (I) (measuring tape)

$L_a \pm \Delta L_a = (628 \pm 1) \times 10^{-3}\,\mathrm{m}$ (0.25) LABEL (I) (measuring tape)

$d = L_b - L_a = (24.6 \pm 0.1) \times 10^{-3}\,\mathrm{m}$ (0.25) LABEL (H) (caliper)





Task 1.4 Performing a statistical and graphical analysis.

1.4 A procedure: 3.25

From the condition of dark fringes and Task 1.2, we have

$$\frac{1}{2}L_R^2(n)\left(\frac{1}{L_0}-\frac{1}{L_b}\right)=\left(n+\frac{5}{8}\right)\lambda$$

and

$$\frac{1}{2}L_L^2(n)\left(\frac{1}{L_a}-\frac{1}{L_0}\right)=\left(n+\frac{7}{8}\right)\lambda$$

Using (1.5), $L_R(n)=l_R(n)-l_{0R}$ and $L_L(n)=l_L(n)-l_{0L}$ we can rewrite

$$\frac{1}{2}\left(l_R(n)-l_{0R}\right)^2\left(\frac{1}{L_0}-\frac{1}{L_b}\right)=\left(n+\frac{5}{8}\right)\lambda$$

$$\Rightarrow \quad l_R(n)=\sqrt{\frac{2L_bL_0}{L_b-L_0}}\lambda\sqrt{n+\frac{5}{8}}+l_{0R}$$

and

$$\frac{1}{2}\left(l_L(n) - l_{0L}\right)^2 \left(\frac{1}{L_a} - \frac{1}{L_0}\right) = \left(n + \frac{7}{8}\right)\lambda$$

$$\Rightarrow \quad l_L(n) = \sqrt{\frac{2L_aL_0}{L_0 - L_a}\,\lambda}\sqrt{n + \frac{7}{8}} + l_{0L}$$

These can be cast as equations of a straight line, $y = mx + b$.

Case I:

$$y_R = l_R \qquad x_R = \sqrt{n + \frac{5}{8}} \qquad m_R = \sqrt{\frac{2L_bL_0}{L_b - L_0}\,\lambda} \qquad b_R = l_{0R}$$

Case II:

$$y_L = l_L \qquad x_L = \sqrt{n + \frac{7}{8}} \qquad m_L = \sqrt{\frac{2L_aL_0}{L_0 - L_a}\,\lambda} \qquad b_L = l_{0L}$$

Perform least squares analysis of above equations. In Table I, we write down the values x_R and x_L.

One finds:

$$m_R \pm \Delta m_R = (-6.39 \pm 0.07) \times 10^{-3}\,\mathrm{m}$$
$$m_L \pm \Delta m_L = (6.83 \pm 0.19) \times 10^{-3}\,\mathrm{m}$$

and (values of l_{0R} and l_{0L})

$$l_{0R} \pm \Delta l_{0R} = b_R \pm \Delta b_R = (-2.06 \pm 0.17) \times 10^{-3}\,\mathrm{m}$$
$$l_{0L} \pm \Delta l_{0L} = b_L \pm \Delta b_L = (-5.33 \pm 0.36) \times 10^{-3}\,\mathrm{m}$$

The equations used in the least squares analysis:

$$m = \frac{N\sum_{n=1}^{N} x_n y_n - \sum_{n=1}^{N} x_n \sum_{n'=1}^{N} y_{n'}}{\Delta}$$
$$b = \frac{\sum_{n=1}^{N} x_n^2 \sum_{n'=1}^{N} y_{n'} - \sum_{n=1}^{N} x_n \sum_{n'=1}^{N} x_{n'} y_{n'}}{\Delta}$$

where

$$\Delta = N\sum_{n=1}^{N} x_n^2 - \left(\sum_{n=1}^{N} x_n\right)^2$$

with N the number of data points. The uncertainty is calculated as

$$(\Delta m)^2 = N\frac{\sigma^2}{\Delta}, \qquad (\Delta b)^2 = \frac{\sigma^2}{\Delta}\sum_{n=1}^{N} x_n^2 \qquad \text{with,}$$

$$\sigma^2 = \frac{1}{N-2}\sum_{n=1}^{N}\left(y_n - b - mx_n\right)^2$$

REFERENCE: P.R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill, 1969.

Task 1.5 Calculating λ.

1.5 From any slope and the value of L_0 one finds, 2.0

$$\lambda = \frac{L_b - L_a}{2L_aL_b}\,\frac{m_R^2 m_L^2}{m_R^2 + m_L^2}$$

Using the suggestion to replace $d = L_b - L_a$, we can write

$$\lambda = \frac{d}{2L_aL_b}\,\frac{m_R^2 m_L^2}{m_R^2 + m_L^2}$$

$$\lambda \pm \Delta\lambda = (663 \pm 25) \times 10^{-9}\,\mathrm{m}$$

The uncertainty may range from 15 to 30 nanometers.

A precise measurement of the wavelength is $\lambda \pm \Delta\lambda = (655 \pm 1) \times 10^{-9}\,\mathrm{m}$.

The formula for the uncertainty,

$$\Delta\lambda = \sqrt{\left(\frac{\partial\lambda}{\partial d}\right)^2 \Delta d^2 + \left(\frac{\partial\lambda}{\partial L_a}\right)^2 \Delta L_a^2 + \left(\frac{\partial\lambda}{\partial L_b}\right)^2 \Delta L_b^2 + \left(\frac{\partial\lambda}{\partial m_R}\right)^2 \Delta m_R^2 + \left(\frac{\partial\lambda}{\partial m_L}\right)^2 \Delta m_L^2}$$

one finds,

$$\frac{\partial\lambda}{\partial d} = \frac{\lambda}{d}\,, \qquad \frac{\partial\lambda}{\partial L_b} = \frac{\lambda}{L_b}\,, \qquad \frac{\partial\lambda}{\partial L_a} = \frac{\lambda}{L_a} \qquad \text{and} \qquad \frac{\partial\lambda}{\partial m_R} = \frac{2m_L^2}{m_R}\,\frac{\lambda}{m_L^2 + m_R^2}$$

and analogously for the other slope.

One can calculate directly these quantities. However, one may note that the errors due to L_a, L_b and d are negligible. Moreover, $m_R^2 \approx m_L^2$ and $L_a \approx L_b$. This implies,

$$\frac{\partial\lambda}{\partial m_R} \approx \frac{\lambda}{m_R} \approx \frac{\partial\lambda}{\partial m_L}$$

. Thus,

$$\Delta\lambda \approx \sqrt{2}\,\frac{\lambda}{m_L}\,\Delta m_L \approx \left(25 \times 10^{-9}\right)\,\mathrm{m}$$