Problem T3. Protostar formation ***(9 points)***

Let us model the formation of a star as follows. A spherical cloud of sparse interstellar gas, initially at rest, starts to collapse due to its own gravity. The initial radius of the ball is r_0 and the mass is m. The temperature of the surroundings (much sparser than the gas) and the initial temperature of the gas is uniformly T_0. The gas may be assumed to be ideal. The average molar mass of the gas is μ and its adiabatic index is $\gamma > \frac{4}{3}$. Assume that $G\frac{m\mu}{r_0} \gg RT_0$, where R is the gas constant and G is the gravitational constant.

i. (0.8 pts) During much of the collapse, the gas is so transparent that any heat generated is immediately radiated away, i.e. the ball stays in thermodynamic equilibrium with its surroundings. What is the number of times, n, by which the pressure increases when the radius is halved to $r_1 = 0.5r_0$? Assume that the gas density remains uniform.

ii. (1 pt) Estimate the time t_2 needed for the radius to shrink from r_0 to $r_2 = 0.95r_0$. Neglect the change of the gravity field at the position of a falling gas particle.

iii. (2.5 pts) Assuming that the pressure remains negligible, find the time $t_{r\to 0}$ needed for the ball to collapse from r_0 down to a much smaller radius, using Kepler's Laws.

iv. (1.7 pts) At some radius $r_3 \ll r_0$, the gas becomes dense enough to be opaque to the heat radiation. Calculate the amount of heat Q radiated away during the collapse from the radius r_0 down to r_3.

v. (1 pt) For radii smaller than r_3 you may neglect heat loss due to radiation. Determine how the temperature T of the ball depends on its radius for $r < r_3$.

vi. (2 pts) Eventually we cannot neglect the effect of the pressure on the dynamics of the gas and the collapse stops at $r = r_4$ (with $r_4 \ll r_3$). However, the radiation loss can still be neglected and the temperature is not yet high enough to ignite nuclear fusion. The pressure of such a protostar is not uniform anymore, but rough estimates with inaccurate numerical prefactors can still be done. *Estimate* the final radius r_4 and the respective temperature T_4.