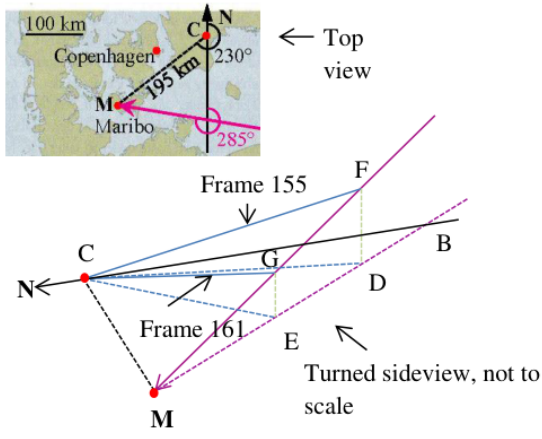
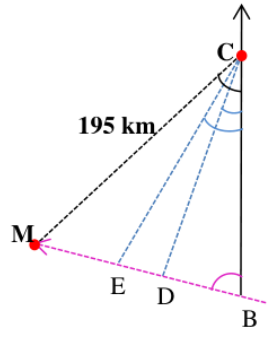
The Maribo Meteorite

Solutions

1.1 1.3





Top view: Triangle MBC: $|\mathrm{CM}| = 195$ km, $\angle\mathrm{BCM} = 230° - 180° = 50°$, and $\angle\mathrm{MBC} = 75°$, so $\angle\mathrm{CMB} = 180° - 75° - 50° = 55°$.

Then $|\mathrm{CB}| = \frac{|\mathrm{CM}|\sin(\angle\mathrm{CMB})}{\sin(\angle\mathrm{MBC})} = 165.4$ km.

Triangle DBC: $|\mathrm{BC}| = 165.4$ km, $\angle\mathrm{BCD} = 215° - 180° = 35°$, and $\angle\mathrm{DBC} = 75°$, so $\angle\mathrm{CDB} = 180° - 75° - 35° = 70°$.

Then $|\mathrm{CD}| = \frac{|\mathrm{BC}|\sin(\angle\mathrm{DBC})}{\sin(\angle\mathrm{CDB})} = 170.0$ km.

Triangle EBC: $|\mathrm{BC}| = 165.4$ km, $\angle\mathrm{BCE} = 221° - 180° = 41°$, and $\angle\mathrm{EBC} = 75°$, so $\angle\mathrm{CEB} = 180° - 75° - 41° = 64°$.

Then $|\mathrm{CE}| = \frac{|\mathrm{BC}|\sin(\angle\mathrm{EBC})}{\sin(\angle\mathrm{CEB})} = 177.7$ km.

Triangle EDC: $\angle\mathrm{DCE} = 41° - 35° = 6°$. Horizontal distance traveled by Maribo: $|\mathrm{DE}| = \frac{|\mathrm{DC}|\sin(\angle\mathrm{DCE})}{\sin(\angle\mathrm{CED})} = 19.77$ km.

Side view: Triangle CDF: $|\mathrm{DF}| = |\mathrm{DC}|\tan(\angle\mathrm{FCD}) = 59.20$ km.

Triangle CEG: $|\mathrm{EG}| = |\mathrm{CE}|\tan(\angle\mathrm{GCE}) = 46.62$ km.

Thus vertical distance travelled by Maribo: $|\mathrm{DF}| - |\mathrm{EG}| = 12.57$ km.

Total distance travelled by Maribo from frame 155 to 161:

$|\mathrm{FG}| = \sqrt{|\mathrm{DE}|^2 + (|\mathrm{DF}| - |\mathrm{EG}|)^2} = 23.43$ km.

The speed of Maribo is $v = \frac{23.43\,\mathrm{km}}{2.28\,\mathrm{s} - 1.46\,\mathrm{s}} = 28.6\,\mathrm{km/s}$.

1.2a 0.7

Newton's second law: $m_\mathrm{M}\frac{\mathrm{d}v}{\mathrm{d}t} = -k\rho_\mathrm{atm}\pi R_\mathrm{M}^2 v^2$ yields $\frac{1}{v^2}\,\mathrm{d}v = -\frac{k\rho_\mathrm{atm}\pi R_\mathrm{M}^2}{m_\mathrm{M}}\,\mathrm{d}t$.

By integration $t = \frac{m_\mathrm{M}}{k\rho_\mathrm{atm}\pi R_\mathrm{M}^2}\left(\frac{1}{0.9} - 1\right)\frac{1}{v_\mathrm{M}} = 0.88$ s.

Alternative solution: The average force on the meteoroid when the speed decreases from v_M to $0.9\,v_\mathrm{M}$ can be estimated to $F_\mathrm{av} = -k\rho_\mathrm{atm}\pi R_\mathrm{M}^2\,(0.95\,v_\mathrm{M})^2$. Using that the acceleration is approximately constant, $a_\mathrm{av} = -k\rho_\mathrm{atm}\pi R_\mathrm{M}^2\,(0.95\,v_\mathrm{M})^2/m_\mathrm{M}$, results in $t = \frac{-0.1\,v_\mathrm{M}}{a_\mathrm{av}} = 0.87$ s.

1.2b 0.3

$\frac{E_\mathrm{kin}}{E_\mathrm{melt}} = \frac{\frac{1}{2}v_\mathrm{M}^2}{c_\mathrm{sm}(T_\mathrm{sm} - T_0) + L_\mathrm{sm}} = \frac{4.2\times10^8}{2.1\times10^6} = 2.1\times10^2 \gg 1.$

1.3a 0.6

$$[x] = [t]^\alpha[\rho_\mathrm{sm}]^\beta[c_\mathrm{sm}]^\gamma[k_\mathrm{sm}]^\delta = [\mathrm{s}]^\alpha[\mathrm{kg\,m^{-3}}]^\beta[\mathrm{m^2\,s^{-2}K^{-1}}]^\gamma[\mathrm{kg\,m\,s^{-3}K^{-1}}]^\delta,$$

so

$$[\mathrm{m}] = [\mathrm{kg}]^{\beta+\delta}[\mathrm{m}]^{-3\beta+2\gamma+\delta}[\mathrm{s}]^{\alpha-2\gamma-3\delta}[\mathrm{K}]^{-\gamma-\delta}.$$

Thus $\beta+\delta=0$, $-3\beta+2\gamma+\delta=1$, $\alpha-2\gamma-3\delta=0$, and $-\gamma-\delta=0$.

From which $(\alpha,\beta,\gamma,\delta)=\left(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2},+\frac{1}{2}\right)$ and $x(t)\approx\sqrt{\frac{k_{\mathrm{sm}}\,t}{\rho_{\mathrm{sm}}c_{\mathrm{sm}}}}$.

1.3b 0.4

$x(5\,\mathrm{s})=1.6$ mm $\quad x/R_{\mathrm{M}}=1.6\,\mathrm{mm}/130\,\mathrm{mm}=0.012$.

1.4a 0.3

Rb-Sr decay scheme: ${}^{87}_{37}\mathrm{Rb}\rightarrow{}^{87}_{38}\mathrm{Sr}+{}^{0}_{-1}\mathrm{e}+\bar{\nu}_{\mathrm{e}}$

1.4b 0.7

$N_{87\mathrm{Rb}}(t)=N_{87\mathrm{Rb}}(0)\,\mathrm{e}^{-\lambda t}$ and Rb→Sr: $N_{87\mathrm{Sr}}(t)=N_{87\mathrm{Sr}}(0)+[N_{87\mathrm{Rb}}(0)-N_{87\mathrm{Rb}}(t)]$.

Thus $N_{87\mathrm{Sr}}(t)=N_{87\mathrm{Sr}}(0)+(\mathrm{e}^{\lambda t}-1)N_{87\mathrm{Rb}}(t)$, and dividing by $N_{86\mathrm{Sr}}$ we obtain the equation of a straight line:

$$\frac{N_{87\mathrm{Sr}}(t)}{N_{86\mathrm{Sr}}}=\frac{N_{87\mathrm{Sr}}(0)}{N_{86\mathrm{Sr}}}+(\mathrm{e}^{\lambda t}-1)\,\frac{N_{87\mathrm{Rb}}(t)}{N_{86\mathrm{Sr}}}.$$

1.4c 0.4

Slope: $\mathrm{e}^{\lambda t}-1=a=\frac{0.712-0.700}{0.25}=0.050$ and $T_{\frac{1}{2}}=\frac{\ln(2)}{\lambda}=4.9\times10^{10}$ year.

So $\tau_{\mathrm{M}}=\ln(1+a)\frac{1}{\lambda}=\frac{\ln(1+a)}{\ln(2)}T_{\frac{1}{2}}=3.4\times10^{9}$ year .

1.5 0.6

Kepler's 3rd law on comet Encke and Earth, with the orbital semi-major axis of Encke given by $a=\frac{1}{2}(a_{\min}+a_{\max})$. Thus $t_{\mathrm{Encke}}=\left(\frac{a}{a_{\mathrm{E}}}\right)^{\frac{3}{2}}t_{\mathrm{E}}=3.30$ year $=1.04\times10^{8}$ s.

1.6a 0.7

For Earth around its rotation axis: Angular velocity $\omega_{\mathrm{E}}=\frac{2\pi}{24\,\mathrm{h}}=7.27\times10^{-5}\,\mathrm{s}^{-1}$.

Moment of inertia $I_{\mathrm{E}}=0.83\,\frac{2}{5}\,m_{\mathrm{E}}R_{\mathrm{E}}^{2}=8.07\times10^{37}$ kg m^2.

Angular momentum $L_{\mathrm{E}}=I_{\mathrm{E}}\omega_{\mathrm{E}}=5.87\times10^{33}$ kg m^{2}s^{-1}.

Asteroid: $m_{\mathrm{ast}}=\frac{4\pi}{3}R_{\mathrm{ast}}^{3}\rho_{\mathrm{ast}}=1.57\times10^{15}$ kg and angular momentum $L_{\mathrm{ast}}=m_{\mathrm{ast}}v_{\mathrm{ast}}R_{\mathrm{E}}=2.51\times10^{26}$ kg m^{2}s^{-1}. L_{ast} is perpendicular to L_{E}, so by conservation angular momentum: $\tan(\Delta\theta)=L_{\mathrm{ast}}/L_{\mathrm{E}}=4.27\times10^{-8}$. The axis tilt $\Delta\theta=4.27\times10^{-8}$ rad (so the North Pole moves $R_{\mathrm{E}}\,\Delta\theta=0.27$ m).

1.6b 0.7

At vertical impact $\Delta L_{\mathrm{E}}=0$ so $\Delta(I_{\mathrm{E}}\omega_{\mathrm{E}})=0$. Thus $\Delta\omega_{\mathrm{E}}=-\omega_{\mathrm{E}}(\Delta I_{\mathrm{E}})/I_{\mathrm{E}}$, and since $\Delta I_{\mathrm{E}}/I_{\mathrm{E}}=m_{\mathrm{ast}}R_{\mathrm{E}}^{2}/I_{\mathrm{E}}=7.9\times10^{-10}$ we obtain $\Delta\omega_{\mathrm{E}}=-5.76\times10^{-14}$ s^{-1}. The change in rotation period is $\Delta T_{\mathrm{E}}=2\pi\left(\frac{1}{\omega_{\mathrm{E}}+\Delta\omega_{\mathrm{E}}}-\frac{1}{\omega_{\mathrm{E}}}\right)\approx-2\pi\frac{\Delta\omega_{\mathrm{E}}}{\omega_{\mathrm{E}}^{2}}=6.84\times10^{-5}$ s.

1.6c 0.7

At tangential impact L_{ast} is parallel to L_{E} so $L_{\mathrm{E}}+L_{\mathrm{ast}}=(I_{\mathrm{E}}+\Delta I_{\mathrm{E}})(\omega_{\mathrm{E}}+\Delta\omega_{\mathrm{E}})$ and thus $\Delta T_{\mathrm{E}}=2\pi\left(\frac{1}{\omega_{\mathrm{E}}+\Delta\omega_{\mathrm{E}}}-\frac{1}{\omega_{\mathrm{E}}}\right)=2\pi\left(\frac{I_{\mathrm{E}}+\Delta I_{\mathrm{E}}}{L_{\mathrm{E}}+L_{\mathrm{ast}}}-\frac{1}{\omega_{\mathrm{E}}}\right)=-3.62\times10^{-3}$ s.

1.7 1.6

Maximum impact speed $v_{\mathrm{imp}}^{\max}$ arises from three contributions:

(I) The velocity v_{b} of the body at distance a_{E} (Earth orbit radius) from the Sun,

$v_{\mathrm{b}}=\sqrt{\frac{2Gm_S}{a_E}}=42.1$ km/s.

(II) The orbital velocity of the Earth, $v_{\mathrm{E}}=\frac{2\pi a_E}{1\,\mathrm{year}}=29.8$ km/s.

(III) Gravitational attraction from the Earth and kinetic energy seen from the Earth:

$$\frac{1}{2}(v_{\mathrm{b}} + v_{\mathrm{E}})^2 = -\frac{Gm_E}{R_E} + \frac{1}{2}\left(v_{\mathrm{imp}}^{\max}\right)^2 .$$

In conclusion: $v_{\mathrm{imp}}^{\max} = \sqrt{(v_{\mathrm{b}} + v_{\mathrm{E}})^2 + \frac{2Gm_E}{R_E}} = 72.8$ km/s.

Total **9.0**