Solar cells (solution)

2.1 The dependence of the solar cell current on the distance to the light source

$$I(r) = \frac{I_a}{1 + \frac{r^2}{a^2}}$$

2.1a Measure I as a function of r, and set up a table of your measurements. 1.0

2.1b Determine the values of I_a and a by the use of a suitable graphical method. 1.0

slot #	r mm	I mA	$1/I$ 1/mA	r^2 mm^2
3	9.0	5.440	0.184	81
4	14.5	5.290	0.189	210
5	20.0	5.010	0.200	400
6	25.5	4.540	0.220	650
7	31.0	3.840	0.260	961
8	36.5	3.230	0.310	1332
9	42.0	2.730	0.366	1764
10	47.5	2.305	0.434	2256
11	53.0	1.985	0.504	2809
12	58.5	1.730	0.578	3422
13	64.0	1.485	0.673	4096
14	69.5	1.305	0.766	4830
15	75.0	1.140	0.877	5625
16	80.5	1.045	0.957	6480
17	86.0	0.930	1.075	7396
18	91.5	0.840	1.190	8372
19	97.0	0.755	1.325	9409
20	102.5	0.690	1.449	10506

$$I\left(1 + \frac{r^2}{a^2}\right) = I_a$$

$$r^2 = I_a a^2 \cdot \frac{1}{I} - a^2$$

$$a^2 = 12 \text{ mm}^2 \pm 100 \text{ mm}^2,$$

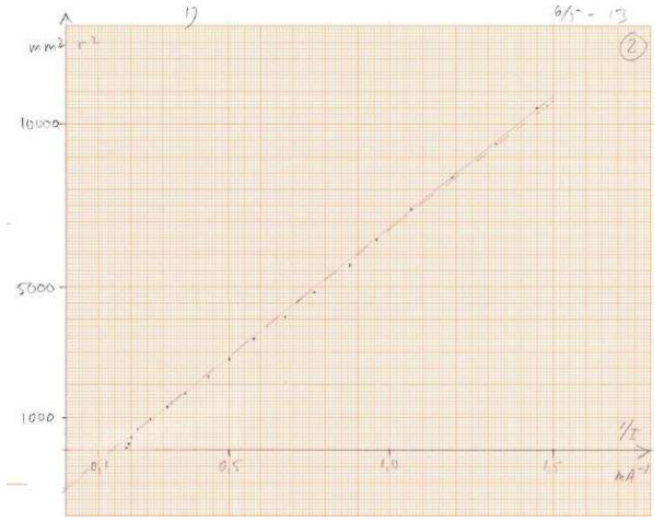
$$a = 35 \text{ mm} \pm \pm 2 \text{ mm}$$

$$I_a a^2 = \frac{10870 - 0}{1.50 - 0.15} \cdot \frac{\text{mm}^2}{\text{mA}^{-1}} = 8051.85 \ \ldots \ \text{m}^2\,\text{mA}$$

$$I_a = \frac{8051.85 \ \frac{\text{mm}^2}{\text{mA}^{-1}}}{1200 \text{ mm}^2} = 6.7 \text{ mA} \pm 0.5 \text{ mA}$$

$$(I_a a^2)_{\min} = \frac{10700 - 0}{1.50 - 0.14} \cdot \frac{\text{mm}^2}{\text{mA}^{-1}} = 7867.6 \ \ldots \ \text{m}^2\,\text{mA}$$

$$\rightarrow I_{a,\max} = \frac{(I_a a^2)_{\min}}{a^2_{\min}} = \frac{7867.6 \text{ mm}^2\,\text{mA}}{1100 \text{ mm}^2} = 7.2 \text{ mA}$$

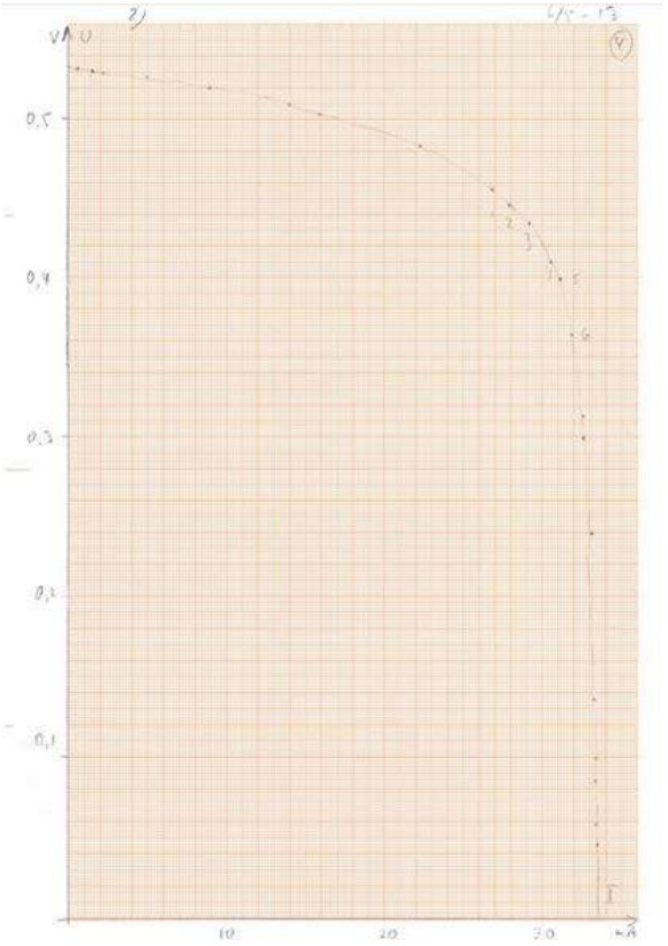


2.2 Characteristic of the solar cell

2.2a Make a table of corresponding measurements of U and I. 0.6

2.2b Graph voltage as a function of current 0.8

I mA	U V
0.496	0.532
1.451	0.531
5.05	0.526
8.88	0.52
14.05	0.509
31.1	0.395
25.3	0.471
21.6	0.488
30.6	0.41
31.9	0.364
32.6	0.299
32.6	0.313
33.1	0.239
33.4	0.085
33.3	0.138
33.4	0.096
33.4	0.058
33.5	0.046
33.5	0.045
1.05	0.529
27.8	0.454
15.9	0.503
22.3	0.483
26.8	0.458
29.2	0.435



2.3 Theoretical characteristic for the solar cell

2.3a Use the graph from question 2.2b to determine $I_{\max}$. 0.4

2.3b Estimate the range of values of U for which the mentioned approximation is good. Determine graphically the values of I_0 and η for your solar cell. 1.2

$$I = I_{\max} \text{ for } U = 0 \;\rightarrow\; I_{\max} = 33.5 \text{ mA}$$

$$\eta k_B T < 4 \cdot 1.381 \cdot 10^{-2} \text{ J/K} \cdot 300 \text{ K} = 0.103 \text{ eV}$$

$$I = I_{\max} - I_0 \left(\exp\left(\frac{eU}{\eta k_B T} \right) - 1 \right) \approx I_{\max} - I_0 \exp\left(\frac{eU}{\eta k_B T} \right)$$

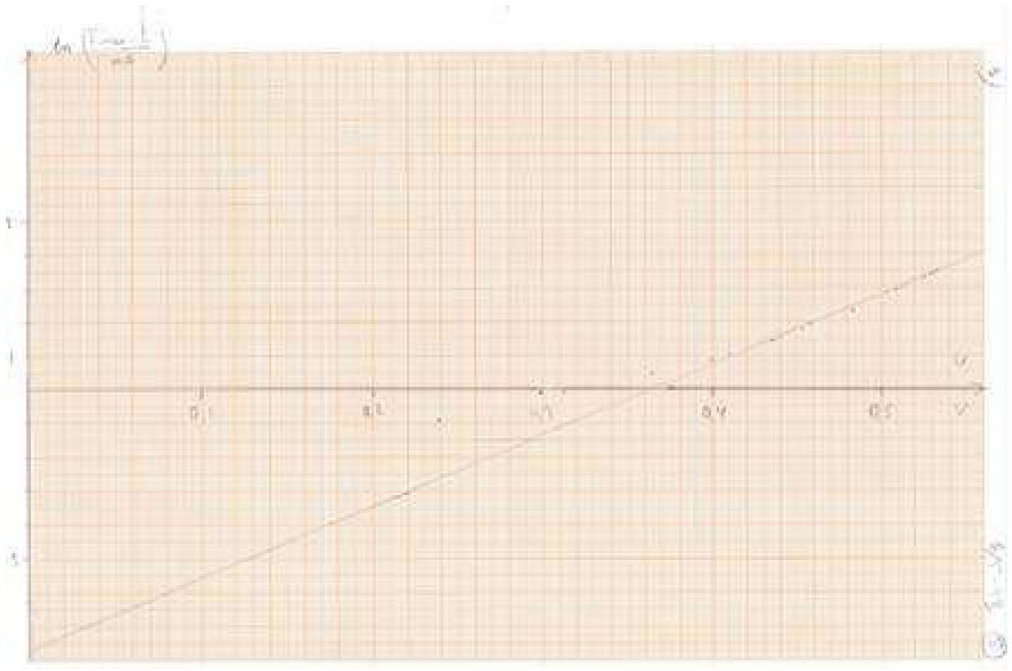
$$\text{for } U > 0.4\,V \text{ where } \exp\left(\frac{eU}{\eta k_B T} \right) > \exp(4) \gg 1$$

$$\ln\left(\frac{I_{\max} - I}{\text{mA}} \right) = \frac{e}{\eta k_B T}\, U + \ln\left(\frac{I_0}{\text{mA}} \right)$$

$$\frac{e}{\eta k_B T} = \frac{4.03 - (-7.7)}{0.56 \text{ V}} = 20.95 \text{ V}^{-1}$$

$$I_0 = e^{-7.7} \text{ mA} = 0.45\,\mu A$$

$$\rightarrow\; \boldsymbol{\eta} = \frac{e/(k_B T)}{20.95 \text{ V}^{-1}} = 1.85$$

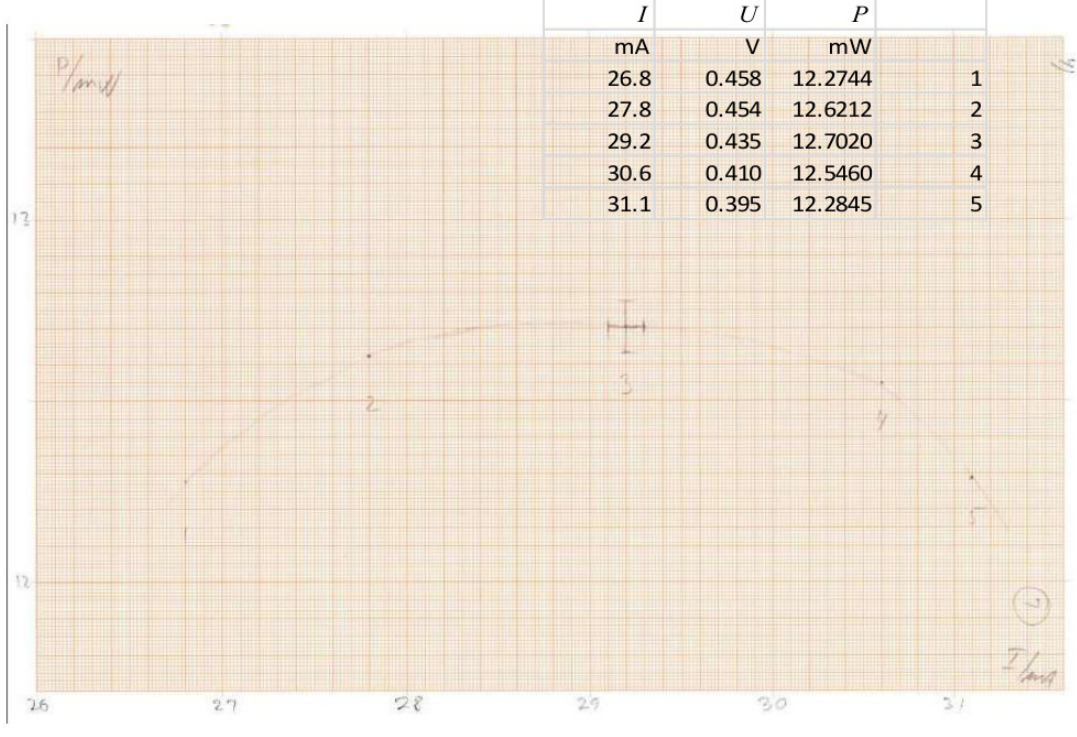


2.4 Maximum power for a solar cell

2.4a The maximum power that the solar cell can deliver to the external circuit is denoted $P_{\max}$. Determine $P_{\max}$ for your solar cell through a few, suitable measurements. (You may use some of your previous measurements from question 2.2) 0.5

2.4b Estimate the optimal load resistance R_{opt}, i.e. the total external resistance when the solar cell delivers its maximum power to R_{opt}. State your result with uncertainty and illustrate your method with suitable calculations. 0.5

I mA	U V	P mW	
26.8	0.458	12.2744	1
27.8	0.454	12.6212	2
29.2	0.435	12.7020	3
30.6	0.410	12.5460	4
31.1	0.395	12.2845	5



$$P_{\max} = (12.7 \pm 0.1)\ \text{mW at } I = (28.8 \pm 0.2)\ \text{mA}$$

$$R_{\text{opt}} = \frac{P_{\max}}{I_{\text{opt}}^2} = \frac{12.71\,\text{mW}}{(28.8\,\text{mA})^2} = (15.3 \pm 0.3)\,\Omega$$

2.5 Comparing the solar cells

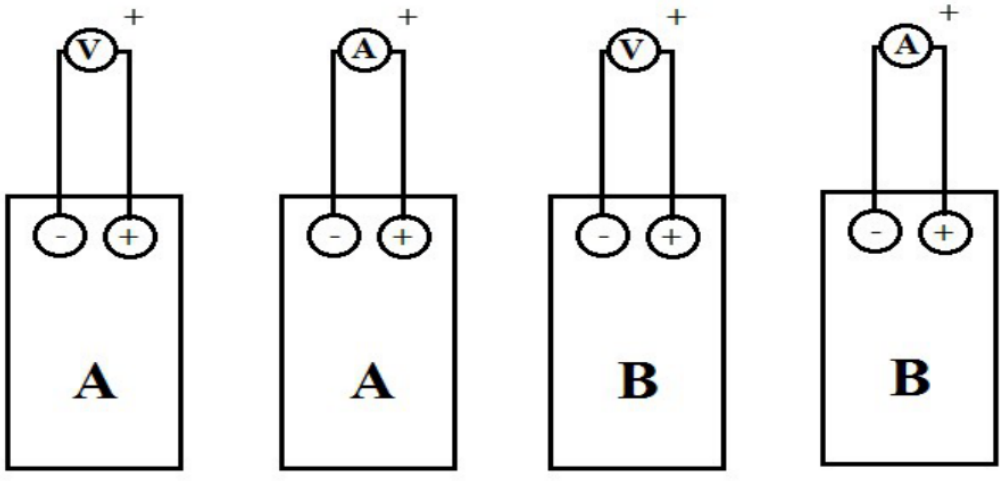
2.5a Measure, for the given illumination:
- The maximum potential difference $U_A$that can be measured over solar cell A.
- The maximum current I_A that can be measured through solar cell A.

Do the same for solar cell B. 0.5

2.5b Draw electrical diagrams for your circuits showing the wiring of the solar cells and the meters. 0.3

2.5a. $U_A = 0.512$ V $\quad I_A = 16.465$ mA $\quad U_B = 0.480$ V $\quad I_B = 16.325$ mA

2.5b.



2.6 Couplings of the solar cells

2.6 Determine which of the four arrangements of the two solar cells yields the highest possible power in the external circuit when one of the solar cells is shielded with the shielding plate (J in Fig. 2.1). Draw the corresponding electrical diagram. 1.0

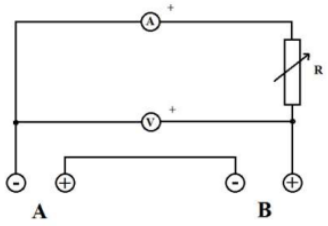
Two approaches:

Approach 1: use a constant setting of the variable resistor to simulate a constant external load.

Approach 2: use the hint given in the question and measure values of maximal U and maximal I independently (no variable resistor involved).

In the following only measurements for approach 1 are presented.

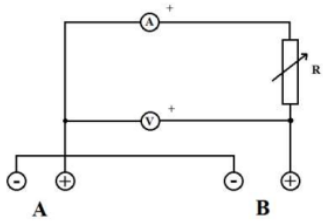
a.



Unshielded (adjusting R for reasonable P)
13.10 mA; 0.794 V; 10.4 mW

A shielded: 0.37 mA; 0.022 V
B shielded: 0.83 mA; 0.049 V

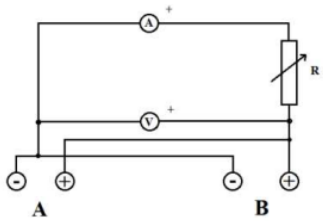
b.



R like in a.

A shielded: 1.47 mA; 0.088 V
B shielded: -2.82 mA; -0.170 V

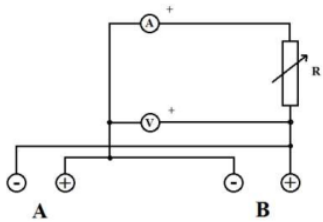
c.



R like in a.

A shielded: 6.89 mA; 0.415 V
B shielded: 6.905 mA; 0.4165 V

d.



R like in a.

A shielded: 7.14 mA; 0.436 V
B shielded: -7.76 mA; -0.474 V

Conclusion: Best power: Set-up d with B shielded. (Solar cell A slightly better than B).

(2.7 on next page)

2.7 The effect of the optical vessel (large cuvette) on the solar cell current

2.7a Measure the current I, now as a function of the height, h, of water in the vessel, see Fig. 2.8. Make a table of the measurements and draw a graph. 1.0

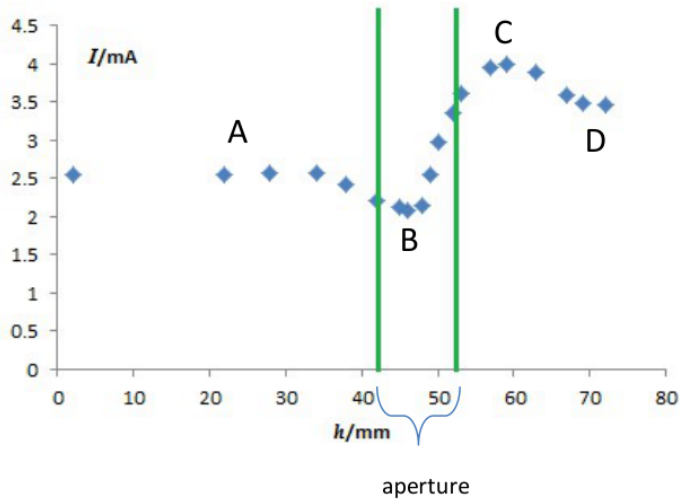
2.7b Explain with only sketches and symbols why the graph looks the way it does. 1.0

2.7c For this set-up do the following: 0.6
- Measure the distance r_1 between the light source and the solar cell, and the current I_1.
- Place the empty vessel immediately in front of the circular aperture and measure the current I_2.
- Fill up the vessel with water, almost to the top, and measure the current I_3.

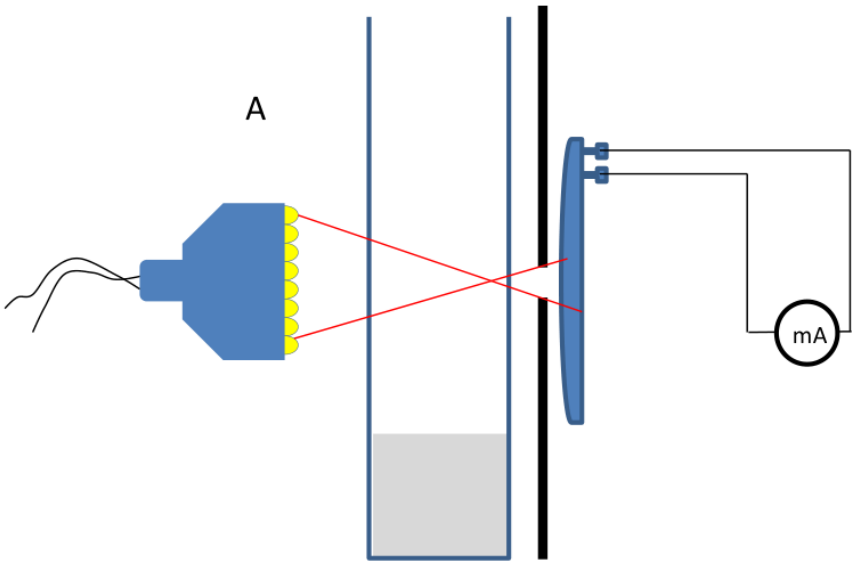
2.7d Use your measurements from 2.7c to find a value for the refractive index n_{w} for water. Illustrate your method with suitable sketches and equations. You may include additional measurements. 1.6

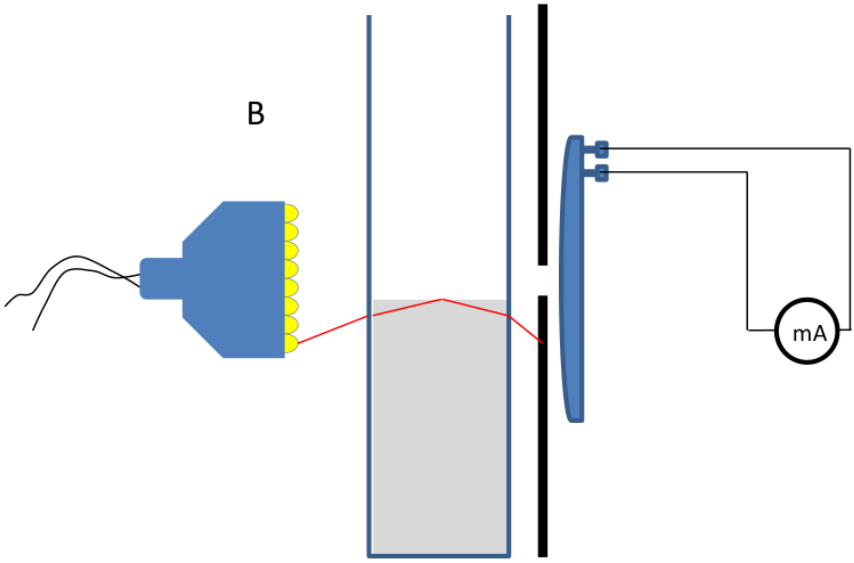
2.7a

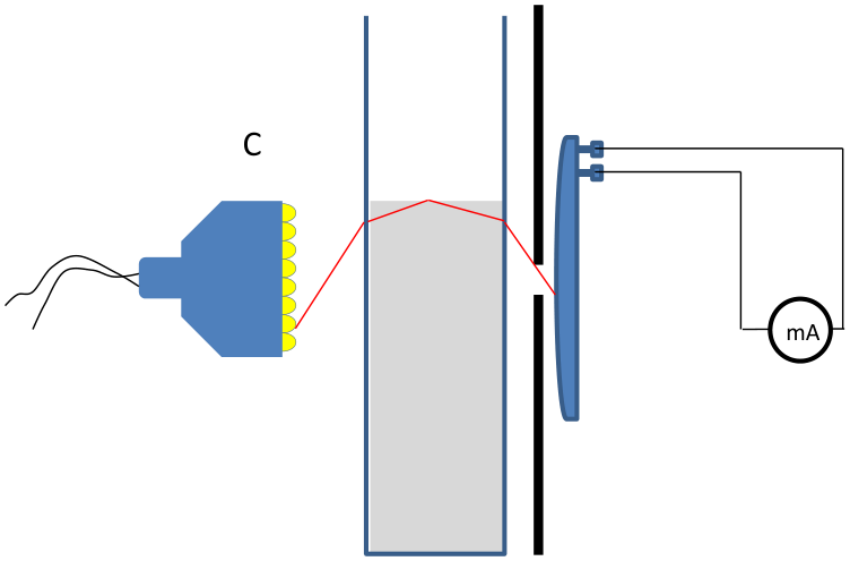
h mm	I mA
2	2.54
22	2.55
28	2.56
34	2.57
38	2.42
42	2.21
45	2.13
46	2.08
48	2.15
49	2.54
50	2.97
52	3.36
53	3.61
57	3.96
59	3.99
63	3.89
67	3.6
69	3.49
72	3.47

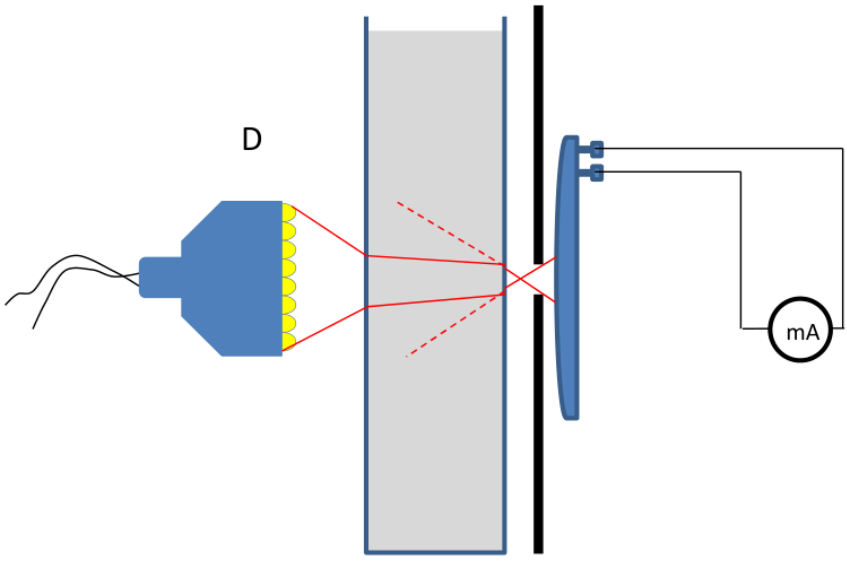


2.7b ***Exemple*** drawings for position A, B, C and D on previous graph:





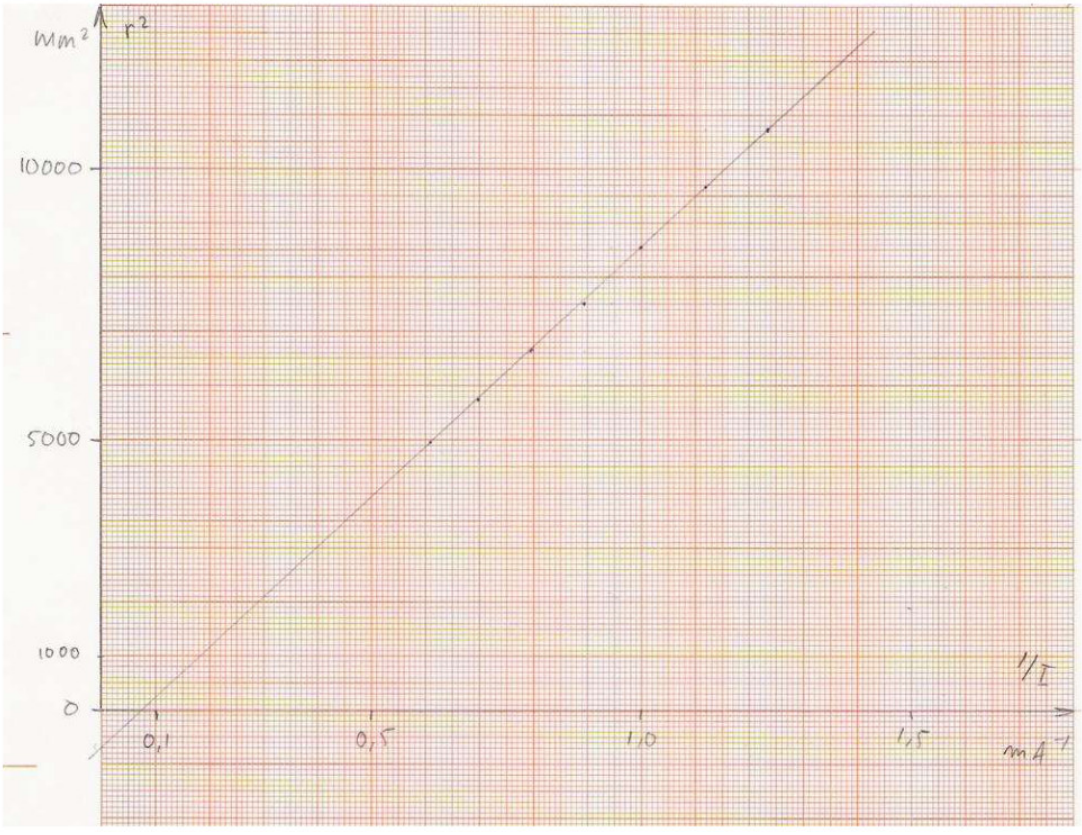




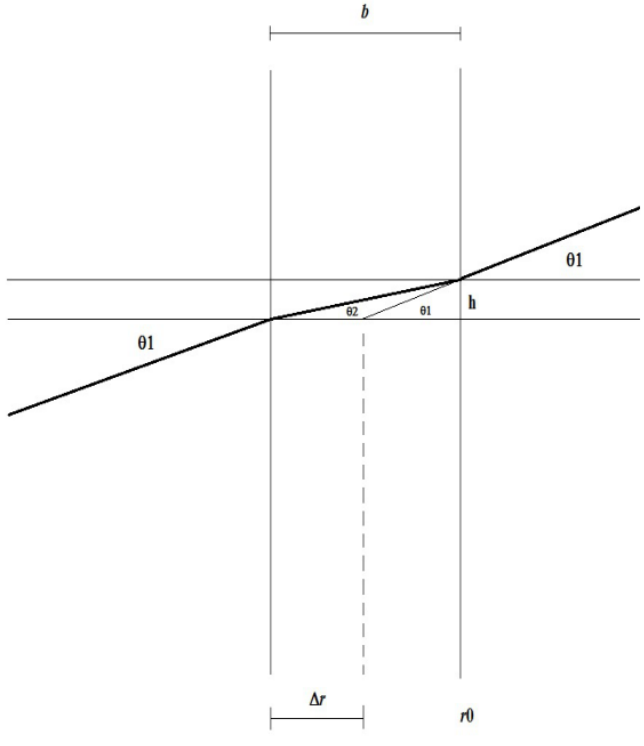
2.7c NOTE: The exemplar measurements are from a different lamp than in 2.1. For a solution to 2.7d using the distance graph it is necessary to refer to the graph below.

$$r_1 = 103.5 \text{ mm}; \quad I_1 = 0.8 \text{ mA}; \quad I_2 = 0.70 \text{ mA}; \quad I_3 = 0.85 \text{ mA}$$

$$\frac{1}{I_3} \cdot \frac{I_2}{I_1} = 1.02 \text{ mA}^{-1} \sim r_c^2 = 880 \text{ mm}^2 \sim r_c = 93.8 \text{ mm}$$



2.7d



$$h = (b - \Delta r)\tan\theta_1 = b\tan\theta_2 \;\Rightarrow\; \frac{b}{b-\Delta r} = \frac{\tan\theta_1}{\tan\theta_2} \approx \frac{\sin\theta_1}{\sin\theta_2} = n, \text{ da } \theta_2 < \theta_1 \ll 1.$$

$$n_{\mathrm{w}} \approx \frac{b}{b-\Delta r} = \frac{b}{b-(r_1 - r_c)} = \frac{26.0\text{ mm}}{26.0\text{ mm} - (103.5 - 93.8)\text{ mm}} = 1.6$$

NOTE: Better results may be obtained. The uncertainty is rather large in this method because of the subtraction of two large numbers for Δr

A different method is to determine the shift by actually moving the set-up and perhaps making an interpolation in directly measured data.