Diffraction due to Helical Structure[1]

(Total marks: 10)

Introduction

The X-ray diffraction image of DNA (Fig. 1) taken in Rosalind Franklin's laboratory, famously known as "Photo 51", became the basis of the discovery of the double helical structure of DNA by Watson and Crick in 1952. This experiment will help you understand diffraction patterns due to helical structures using visible light.

Objective

To determine geometrical parameters of helical structures using diffraction.

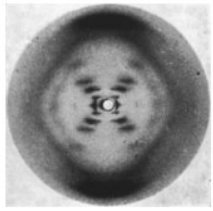
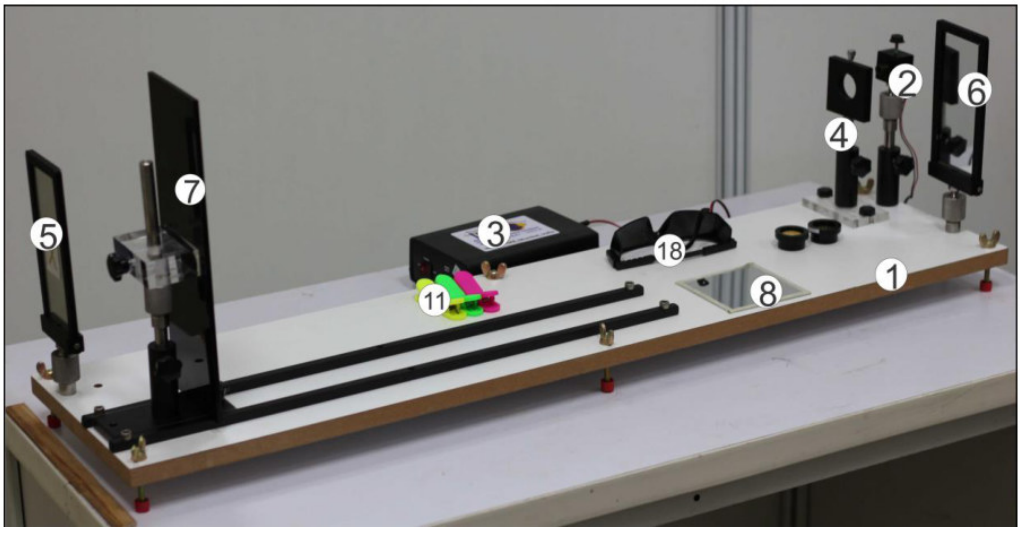


Figure 1: Photo 51



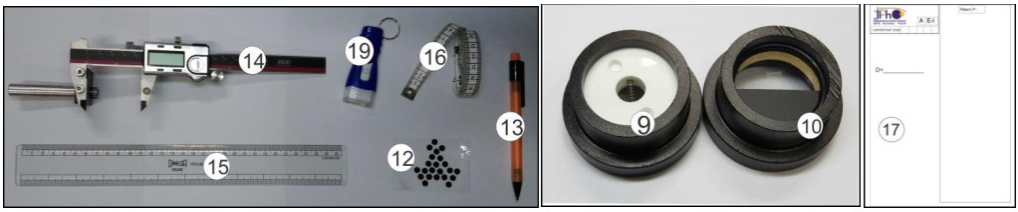


Figure 2: Apparatus for E-I

List of apparatus

[1] *Praveen Pathak (HBCSE-TIFR, Mumbai), Charudatt Kadolkar (IIT, Guwahati), and Manish Kapoor (Christ Church College, Kanpur) were the principal authors of this problem. The contributions of the Academic Committee, Academic Development Group and the International Board are gratefully acknowledged.*

[1]	Wooden platform	[11]	Plastic clips
[2]	Laser source with its mount and base	[12]	Circular black s
[3]	DC regulated power supply for the Laser source	[13]	Mechanical per
[4]	Sample holder with its base	[14]	Digital caliper
[5]	Left reflector (front coated mirror)	[15]	Plastic scale (3
[6]	Right reflector (front coated mirror)	[16]	Measuring tape
[7]	Screen (10 cm x 30 cm) with its mount and base	[17]	Pattern markin
[8]	Plane mirror (10 cm x 10 cm)	[18]	Laser safety go
[9]	Sample I (helical spring)	[19]	Battery operate
[10]	Sample II (double-helix-like pattern printed on glass plate)		

Note: Items [1], [3], [14], [15], [16] and [18] are also used in experiment E-II.

Description of apparatus

Wooden platform [1]: A pair of guiding rails, laser, reflectors, screen and sample mounts are rigidly fixed on it.

Laser source with its mount and base [2]: Laser source of wavelength $\lambda = 635\,\text{nm}$ ($1\ \text{nm} = 10^{-9}\,\text{m}$) is fixed in a metallic mount clamped to the base using a ball joint ([20] in Fig. 3) allowing the adjustment in X-Y-Z directions. The laser body can be rotated and clamped using the top lock-in screw. The beam focus can be adjusted by rotating the front lens cap (red arrow in Fig. 3) to obtain a clear and sharp diffraction pattern.

DC regulated power supply [3]: The front panel has an intensity switch (high/low), socket for laser source connector and three USB sockets. The back panel has power switch and mains power socket (inset of Fig. 4).

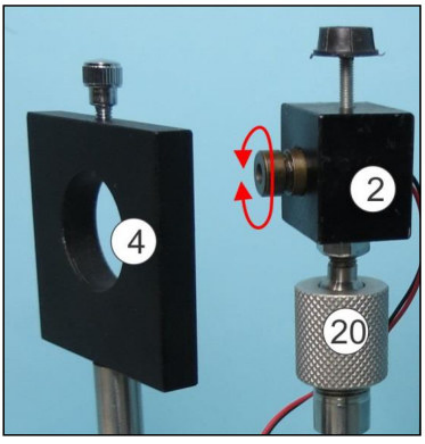


Figure 3: Laser source and sample holder. [20] Ball joint.



Figure 4: DC regulated power supply

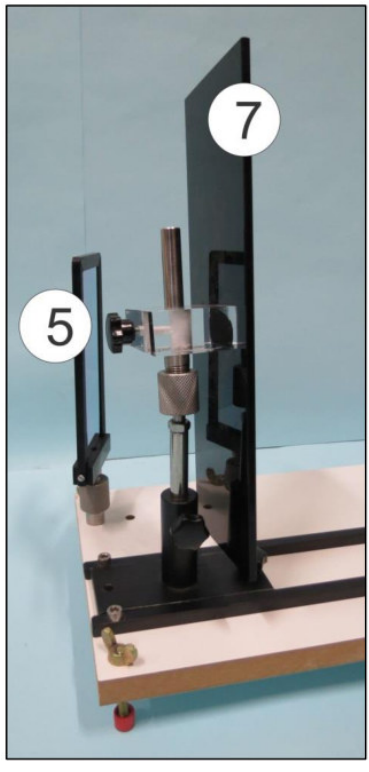


Figure 5: Left reflector and screen

Sample holder with its base [4]: Use the top locking screw to affix the samples in it (Fig. 3). The sample holder can be adjusted horizontally, vertically and rotated.

Left reflector [5]: This reflector is fixed to the platform (Fig. 5). Do not use the side marked X.

Right reflector [6]: This reflector is fixed to the platform and is removable (It will be removed in experiment E-II). Do not use the side marked X.

Screen with its mount [7]: The screen is mounted on ball joint and a base allowing rotational adjustments in all directions (Fig 5). The screen can be fixed as shown in Fig. 2 or Fig. 6 as required.

Sample I [9]: A helical spring fixed on a circular mount using white acrylic plates.

Sample II [10]: A double-helix-like pattern printed on a glass plate which is fixed on a circular mount.

Digital caliper with a mount [14]: Digital caliper is fixed to a mount (the mount is required in E-II). It has an On/Off switch, a switch to reset the reading to zero, a mm/inch selector (keep on mm), a locking screw and a knob for moving the right jaw. The digital caliper can be used to make measurements on pattern marking sheets.

Pattern marking sheets [17]: The given pattern marking sheets can be folded in half and clipped onto the screen using the plastic clips. Ensure that you mark the diffraction pattern within the rectangular box.

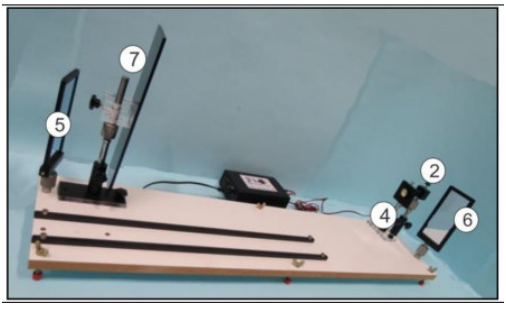


Figure 6: Alternate position of screen compared to that shown in Fig. 2

Theory

A laser beam of wavelength λ, falling normally on a cylindrical wire of diameter a, is diffracted in the direction perpendicular to the wire. The resulting intensity pattern as observed on a screen is shown in Fig. 7.

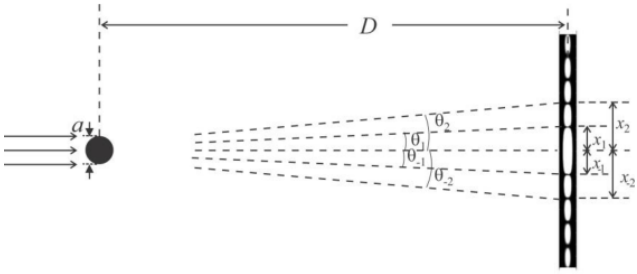


Figure 7: Schematic of the diffraction pattern due to single cylindrical wire of diameter a.

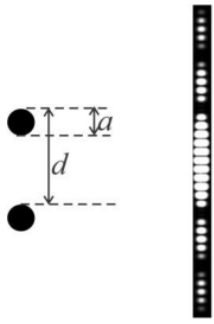


Figure 8: Schematic of diffraction pattern due to two cylindrical wires

The intensity distribution as a function of angle θ with the incident direction is given by

$$I(\theta) = I(0)\left[\frac{\sin\beta}{\beta}\right]^2 \qquad \text{where } \beta = \frac{\pi a \sin\theta}{\lambda}$$

The central spot is bright and for other angles, when $\sin\beta$ ($\beta \neq 0$) is zero, the intensity vanishes. Thus the intensity distribution has n^{th} minimum at the angle θ_n, given by

$$\sin\theta_n = \pm n\frac{\lambda}{a} \qquad n = 1, 2, 3, 4, 5 \ldots.$$

Here $\pm$ refers to both sides of the central spot ($\theta = 0$).

The diffraction pattern due to two parallel identical wires kept at a distance d from each other (Fig. 8) is a combination of two patterns (diffraction due to a single wire and interference due to two wires). The resultant intensity distribution is given by,

$$I(\theta) = I(0)\cos^2\delta\left[\frac{\sin\beta}{\beta}\right]^2$$

where $\delta = \frac{\pi d \sin\theta}{\lambda}$ and $\beta = \frac{\pi a \sin\theta}{\lambda}$

For a screen placed at a large distance D from the wire, the positions of the minima on the screen are observed at $x_{\pm n} = \pm n\frac{\lambda D}{a}$ due to diffraction and at $x_{\pm m} = \pm\left(m - \frac{1}{2}\right)\frac{\lambda D}{d}$ due to interference (where $m, n = 1, 2, 3, 4, 5 \ldots$). Similarly for a set of four identical wires (Fig. 9), the net intensity distribution is a combination of diffraction from each wire and interference due to pairs of wires and hence depends on a, d and s. In other words, the combination of three different intensity patterns is observed.

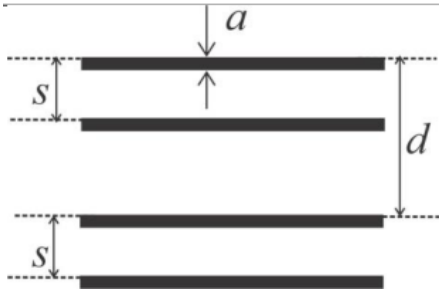


Figure 9: Set of four wires

Initial adjustments

1. Switch on the laser source and adjust both reflectors so that the laser spot falls on the screen.

2. Use the plastic scale and adjust the laser mount and reflectors such that the laser beam is parallel to the wooden platform.

3. Make sure that the laser spot falls near the centre of the screen.

4. Switch off the laser source. Clamp the pattern marking sheet on the screen.

5. Clamp the given plane mirror on the screen using plastic clips and switch on the laser again.

6. Adjust the screen so that the laser beam retraces its path back to the laser source. Remove the mirror once your alignment is completed.

7. Lights in the cubicle may be switched on/off as required.

Experiment

Part A: Determination of geometrical parameters of a helical spring[2]

Sample I is a helical spring of radius R and pitch P made of a wire of uniform thickness a_1 as shown in Fig. 10(a). When viewed at normal incidence its projection is equivalent to two sets of parallel wires of the same thickness separated by distance d_1 and angle $2\alpha_1$ between them (Fig. 10(b)).

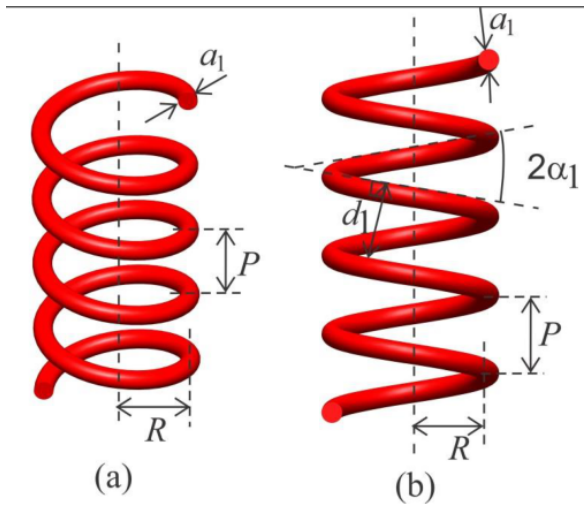


Figure 10: (a) Typical view of helical spring (b) Schematic diagram when viewed at normal incidence

- Mount sample I in the sample holder ensuring that the spring is vertical.
- Obtain a clear and sharp X-shaped diffraction pattern on the pattern marking sheet.
- For this you may adjust

  - laser beam focus (rotate lens cap)

  - beam orientation (rotate the laser body so that only two turns of the spring are illuminated)

  - laser intensity (high/low switch on power supply)

  - ambient light (by switching on or off cubicle light)

If the central maximum is very bright, you may stick circular black stickers on the pattern marking sheet to reduce scattering.

[2]Reference: G. Braun, D. Tierney and H. Schmitzer, Phys. Teach. **49**, 140 (2011).

Tasks	Description	Marks
A1	Mark the appropriate positions (using given pencil [13]) of the intensity minima to determine a_1 and d_1 on the both sides of the central spot on the pattern-marking sheet. Please label your pattern-marking sheets as P-1, P-2 etc.	**0.7**
A2	Measure the appropriate distances using digital calipers and record them in Table A1 for determining a_1.	**0.5**
A3	Plot a suitable graph, label it Graph A1 and from the slope, determine a_1.	**0.7**
A4	Measure the appropriate distances and record them in Table A2 for determining d_1.	**0.8**
A5	Plot a suitable graph, label it Graph A2 and from the slope, determine d_1.	**0.6**
A6	From the X-shaped pattern, determine the angle α_1.	**0.2**
A7	Express P in terms of d_1 and α_1 and calculate P.	**0.2**
A8	Express R in terms of P and α_1 and calculate R (neglect a_1).	**0.2**

Part B: Determination of geometrical parameters of double-helix-like pattern

Figure 11(a) shows two turns of a double helix. Fig. 11(b) is a two-dimensional projection of this double helix when viewed at normal incidence. Each helix of thickness a_2 has an angle $2\alpha_2$ and perpendicular distance d_2 between turns. The separation between two helices is s. Sample II is a double-helix-like pattern printed on glass plate (Fig. 12), whose diffraction pattern is similar to that of a double helix. In this part, you will determine the geometrical parameters of sample II.

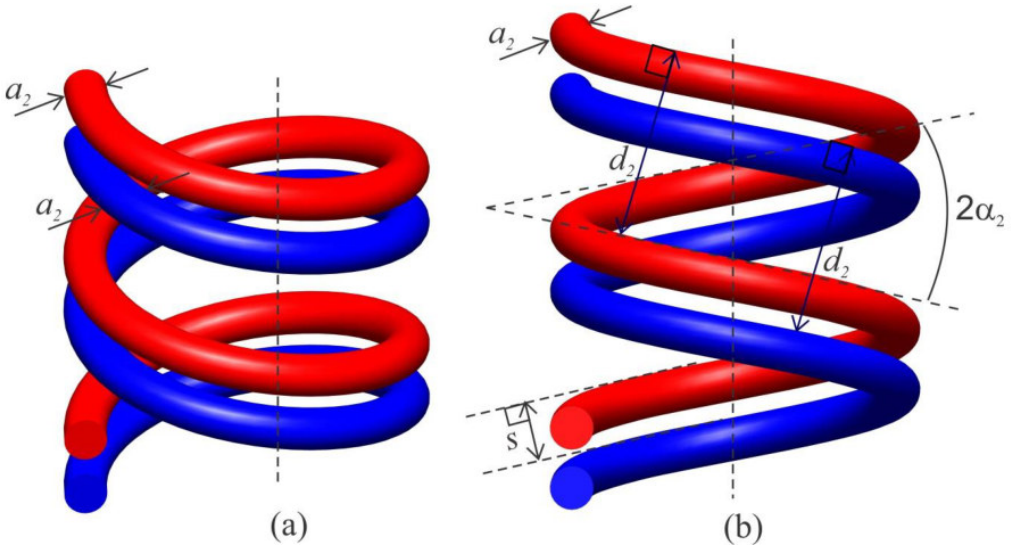


Figure 11: (a) Typical view of double-helical spring (b) Its schematic diagram when viewed at normal incidence.

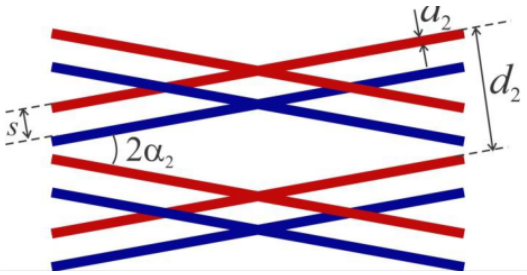


Figure 12: Double-helix-like pattern of sample II

- Mount the sample II in sample holder.

- Attach a new pattern-marking sheet on the screen.
- Obtain clear and sharp X-shaped diffraction pattern on the screen.

Tasks	Description	Marks
B1	Mark the appropriate positions of the minima on either side of the central spot to determine a_2, s and d_2. You can use more than one pattern marking sheets.	**1.1**
B2	Measure the appropriate distances and record them in Table B1 for determining a_2	**0.5**
B3	Plot suitable graph, label it Graph B1 and from the slope, determine a_2.	**0.5**
B4	Measure the appropriate distances and record them in Table B2 for determining s.	**1.2**
B5	Plot suitable graph, label it Graph B2 and from the slope determine s.	**0.5**
B6	Measure the appropriate distances and record them in Table B3 for determining d_2	**1.6**
B7	Plot suitable graph, label it Graph B3 and from the slope, determine d_2.	**0.5**
B8	From the X-shaped pattern, determine the angle α_2.	**0.2**