James Webb Space Telescope (12 points)

This is a question on the physics of the James Webb Space Telescope. Light from a star strikes the primary mirror, with an area of $A_{\text{mirror}} = 25\,\text{m}^2$, and reflects off of a secondary mirror. The focal length of the system is $f = 130\,\text{m}$. The light is focused into the ISIM (Integrated Science Instrument Module), which contains the CCD (charged-coupled device) cameras.

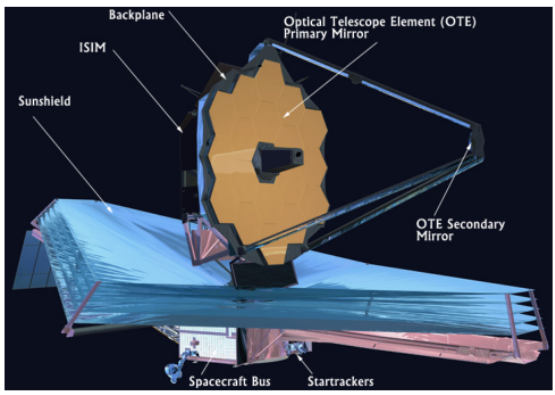


Image Credit: NASA

Part A. Imaging a Star (1.8 points)

The nearest Red Giant is 89 light-years distant, has a temperature of $T_{\text{star}} = 3600\,\text{K}$, and a diameter of $d_o = 1.7 \times 10^{11}\,\text{m}$.

A.1 Calculate the diameter of a focused image of the star on the CCD camera imaging surface. 0.4pt

A.2 Estimate the diameter of a diffraction central maximum on the CCD camera imaging surface. Assume a wavelength of $\lambda = 800\,\text{nm}$, which is the strongest intensity wavelength from the red giant star. 0.4pt

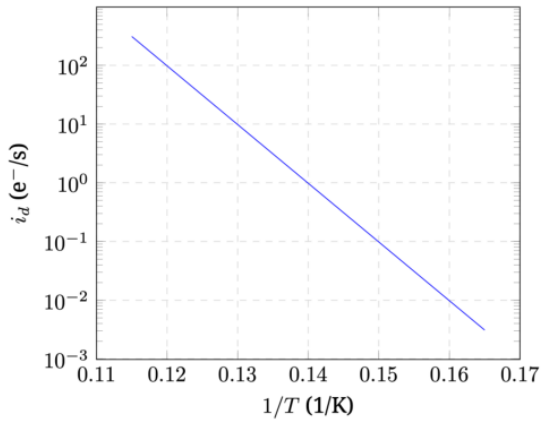
A.3 If the CCD is not cooled and can lose heat only by radiating from the top of the imaging surface, what would be the equilibrium temperature of the CCD at the location of the image of the red giant star? Assume the CCD surface is a blackbody. Provide a formula and a numerical estimate. 1.0pt

Part B. Counting Photons (1.8 points)

The absorption of a photon by the CCD camera leads to the emission of an electron within the apparatus. This occurs only if the photon has sufficient energy to excite an electron across an energy gap ΔE_g. Assume that every photon with sufficient energy succeeds. There is also leakage of electrons across the gap caused by the temperature of the CCD camera; this is the dark current i_d and is measured in the number of electrons per second. It is a function of temperature according to

$$i_d = i_0 e^{-|\Delta E_g|/6k_B T}. \tag{1}$$

where i_0 is a constant.



The graph shows how dark current varies with temperature. The units for dark current, e^-/s should be thought of as counting a number of electrons per second.

B.1 From the dark current graph, provide an order of magnitude estimate for the temperature of a distant source of thermal photons that would just be capable of exciting an electron on the pixel. 0.4pt

The electrons are collected in a capacitor, and after an exposure time τ, the electrons are counted. There are three main sources of uncertainty in the process: a fixed uncertainty in the counting process called read out noise; a Poisson distribution error associated with the dark current, and a Poisson distribution error associated with the detected incoming photons. Poisson distribution errors are equal to the square root of the number of counts associated with a process. The measured photon count is equal to the number of electrons in the capacitor, minus the number of electrons associated with the dark current.

B.2 Write an expression for the total count uncertainty σ_t, if there is a readout noise σ_r, a dark current i_d, an incoming photon rate p, and an exposure time τ. 0.4pt

For remaining questions in this part assume the exposure time is $\tau = 10^4$ s and the read out noise is a fixed $\sigma_r = 14$.

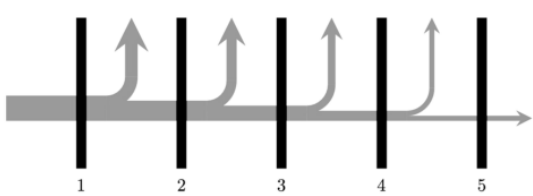
B.3 Assume an operating temperature of $T_\mathrm{p} = 7.5\,\mathrm{K}$. Calculate the minimum photon rate p so that the photon count is ten times the count uncertainty. 0.5pt

B.4 Assuming all photons are just capable of exciting an electron across the band gap, what is the intensity of the source of photons found in B.3 on the primary mirror? Express your answer in $\mathrm{W/m^2}$ 0.5pt

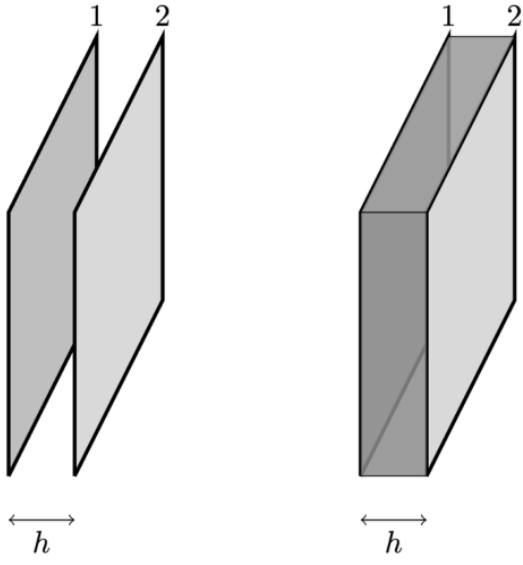
Part C. Passive Cooling (4.4 points)

An infrared CCD camera must be kept at a low temperature. The first tool is a shield to protect from the sun's radiation.

The sun-shield consists of five separated reflective layers in thin sheets (black); radiant energy (gray) from the sun is incident on the first sheet on left, and some energy escapes between every pair of sheets.



Schematic of energy flow: the vertical lines (black) are the sheets, the flow of energy (gray) is from the left to the right, however, between sheets, some energy flows up and out into space.



On the left is a simple model of two adjacent sheets 1 and 2 separated by a distance h. The sheets are not connected, and the perimeter is open to space. Assume the sheets are parallel. Thermal radiation can be exchanged between the sheets, and thermal radiation can escape through the perimeter gap. On the right, the perimeter gap has been shaded to help visualize.

Assume the following simplifications:

- Sheets are square, each with area $A_{\text{sheet}} = 200\,\text{m}^2$.
- Sheets are parallel and separated by $h = 25\,\text{cm}$ along the perimeter.
- Sheets have constant emissivity $\epsilon \ll 1$. Assume that all reflections off of sheet surfaces are diffuse.
- Sheets are thin with temperature on the front and back surfaces equal and uniform.
- The fraction of radiant flux emitted by a sheet that is absorbed by the adjacent sheet is $\alpha \leq 1$. This means that if sheet 1 in the figure above emits an amount of heat Q_1 toward sheet 2 then sheet 2 will absorb an amount αQ_1 from sheet 1.
- The amount of radiant flux ejected out of the perimeter gap between two sheets is approximated as βQ_{12} where αQ_{12} is the net flux between the two sheets. The fraction $\beta < 1$. This is equivalent to saying that the heat loss to space between two sheets is proportional to the net heat exchange between the sheets. This is a rough approximation for this problem.
- Background temperature of space is negligible.

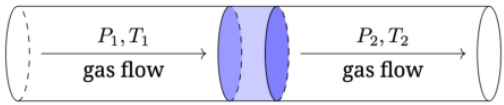
C.1 Derive expressions for the equilibrium temperatures of the first sheet and fifth sheet in terms of the incident solar radiation intensity I_0, the constants α and β, and any necessary physical constants. To simplify your expression, you may define additional constants in terms of α and β, etc. 2.4pt

C.2 Derive numerical estimates for α and β from the information about the sheet geometry assuming an emissivity $\epsilon = 0.05$. You are encouraged to consider the rectangular box model of the sheets above, where the perimeter area effectively acts as a perfect absorber of radiant energy. 1.6pt

C.3 Numerically determine the temperatures of sheet 1 and sheet 5. The solar intensity is $I_0 = 1360\,\text{W/m}^2$. 0.4pt

Part D. Cryo-cooler (4 points)

The last stage of the cooling system directly cools the CCD camera. A closed cycle refrigeration system has a supply pipe line feeding helium gas at constant pressure P_1 moving through a sponge like porous plug into a pipe with constant pressure P_2. The pipe carries the gas to cool the CCD. The helium gas then passes through a pump before returning to the supply line.



Helium gas supplied on the left at well defined pressure P_1 and temperature T_1 is forced through the plug to well defined pressure P_2 and temperature T_2, where it is carried away on the right.

As the gas moves through the porous plug, viscous friction with the narrow walls of the channels in the sponge becomes an important effect; however, no heat is transferred to or from the gas during the process. The bulk speed of the gas in region 2 is only marginally greater than the bulk speed in region 1.

Helium is not an ideal gas, but does remain in a gaseous state throughout this process.

D.1 Consider a mole of gas that passes from left to right through the plug. Complete the table in your answer sheet by writing '>' or '<' to identify the quantity that must be greater, '=' to identify quantities that must be equal, or '?' if it is not possible to know which is greater or equal without more information. 1.0pt

D.2 Identify a conserved quantity constructed from U (internal energy), P (pressure), and V (volume) as a mole of gas moves through the plug; show work on how you derived this conserved quantity. 0.6pt

Your answer sheets have graphs of internal energy per mass against volume per mass for helium with isotherms and lines of constant entropy.

D.3 Assuming that $V_2 = 0.100\,\mathrm{m^3/kg}$ and $T_2 = 7.5\,\mathrm{K}$, use the graph to find a numerical value for the conserved quantity that you found in Part D.2. Show the construction on the graph! 1.4pt

D.4 Find the maximum possible temperature for T_1. Show the construction on the graph! 0.8pt

D.5 Assuming your value for the maximum T_1 found in D.4, find a numerical value for P_1. 0.2pt