E2: Cylindrical Diode - SOLUTION

Take the logarithm of Equation 1,

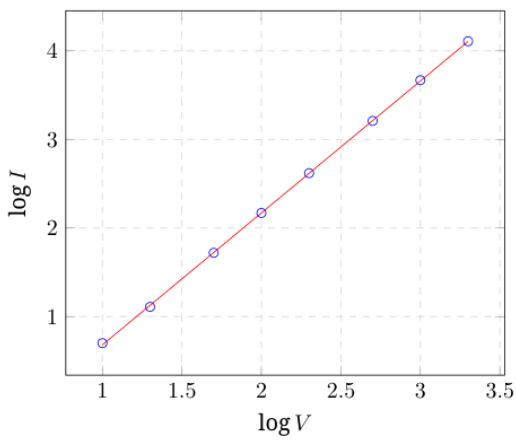
$$\log I_\infty = \log C + \alpha \log R_c + \beta \log L_e + \gamma \log V$$

A.1: Collect data by varying V. To minimize error, select maximum values for all fixed variables, this means $L_e = 99$ cm, $R_c = 10$ cm, and $R_e = 1.0$ cm. Distribute the voltages logarithmically between 10 and 2000

V (V)	I (mA)	$\log V$	$\log I$
10	5	1.0	0.70
20	13	1.3	1.11
50	52	1.7	1.72
100	147	2.0	2.17
200	415	2.3	2.62
500	1620	2.7	3.21
1000	4630	3.0	3.67
2000	12900	3.3	4.11

Plot this on a graph; the best fit line is

$$\log I = 1.490 \log V - 0.8095$$



so $\gamma = 1.49$.

A statistical analysis of the uncertainty in the slope yields $\gamma = 1.490 \pm 0.005$.

Assessing the slope by visually fitting lines through the error bars on the points requires considering that error bars on a log axis are given by

$$\delta(\log y) = \delta\left(\frac{\ln y}{\ln 10}\right) = \frac{1}{\ln 10}\frac{\delta y}{y}$$

Since the largest relative error is in the smallest valued quantity, the focus is on $\delta V/V$ for $V = 10$V and $\delta I/I$ for $I = 5$mA. The error bars associated with the log-log plot at that point are then

$$(1 \pm 0.02, 0.70 \pm 0.04)$$

The other error bars are smaller; focusing on that point alone we can fit two extreme lines and get

$$\gamma = 1.485 \pm 0.025$$

Either approach is acceptable.

Marking scheme:

Data	vary only V	0.05 pts
	$R_e \geq 1$cm	0.05 pts
	$R_c \geq 10R_e$cm	0.05 pts
	$L_e \geq 90$cm	0.05 pts
	table has units	0.05 pts
	V distributed as log	0.05 pts
	$V_{\max} \geq 1000$V	0.05 pts
	$V_{\min} \geq 10$V	0.05 pts
	$V_{\min} \leq 50$V	0.05 pts
	Correct calculations	0.05 pts
	7 or more points	0.30/0.30
	6 points	0.25/0.30
	5 points	0.20/0.30
	4 or fewer points	0.10/0.30
Plotting	covers $> 50\%$ of area	0.10 pts
	Axis labels	0.05 pts
	Axis units correct	0.05 pts
	one plotting mistake	-0.05/-0.10
	two or more plotting mistakes	-0.10/-0.10
Fit	line drawn on graph	0.10 pts
	slope correctly computed with units	0.10 pts
	$1.45 < \gamma < 1.55$	0.10 pts
	uncertainty of slope computed	0.10 pts
	$\delta\gamma \leq 0.03$	0.10 pts
	sum	**1.5 pts**

Measured data should be entered into spreadsheet that will calculate results; if deviation is too large, data point should not count.

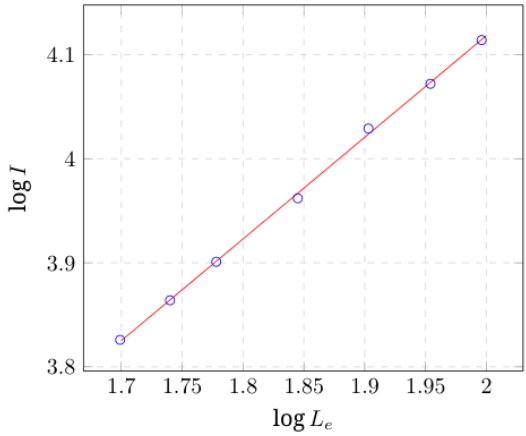
Evidence of reverse engineering should result in zero points for the entire section

A.2: Collect data by varying L_e. To minimize error, select maximum values for all fixed variables, this means $V = 2000$ V, $R_c = 10$ cm, and $R_e = 1$ cm.

L_e (cm)	I (mA)	$\log L_e$	$\log I$
99	13000	1.996	4.144
90	11800	1.954	4.072
80	10700	1.903	4.029
70	9170	1.845	3.962
60	7960	1.778	3.901
55	7310	1.740	3.864
50	6700	1.699	3.826

Plot this on a graph; the best fit line is

$$\log I = 0.9767 \log L_e + 2.1649$$



so $\beta = 0.9767$.

A statistical analysis of the uncertainty in the slope yields $\beta = 0.98 \pm 0.02$.

Graphical fitting of the steepest and shallowest lines yields $\beta = 0.97 \pm 0.02$.

Marking scheme:

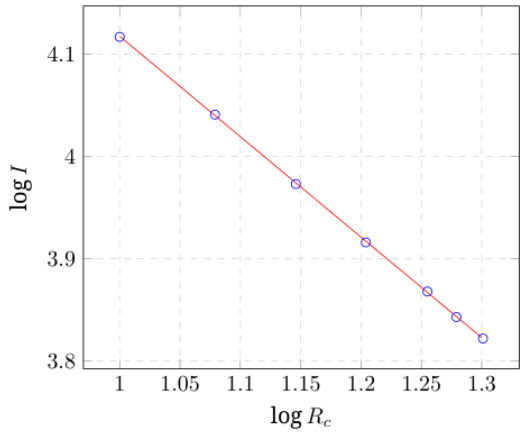
Data	vary only L_e	0.05 pts
	$R_e \geq 1$cm	0.05 pts
	$R_c \geq 10R_e$cm	0.05 pts
	$V \geq 100$V	0.05 pts
	table has units	0.05 pts
	L_e distributed evenly	0.05 pts
	$L_{e,\max} \geq 90$cm	0.05 pts
	$L_{e,\min} \geq 3R_c$	0.05 pts
	$L_{e,\min} \leq 50$cm	0.05 pts
	Correct calculations of derived quantities	0.05 pts
	7 or more points	0.30/0.30
	6 points	0.25/0.30
	5 points	0.20/0.30
	4 or fewer points	0.10/0.30
Plotting	covers $> 50\%$ of area	0.10 pts
	Axis labels	0.05 pts
	Axis units correct	0.05 pts
	one plotting mistake	-0.05/-0.10
	two or more plotting mistakes	-0.10/-0.10
Fit	line drawn on graph	0.10 pts
	slope correctly computed with units	0.10 pts
	$0.97 < \beta < 1.03$	0.10 pts
	uncertainty of slope computed	0.10 pts
	$\delta\beta \leq 0.03$	0.10 pts
	sum	**1.5 pts**

A.3: Collect data by varying R_c. To minimize error, select maximum values for all fixed variables, this means $V = 2000$ V, $L_e = 99$ cm, and $R_e = R_c/10$ cm.

R_c (cm)	I (mA)	$\log R_c$	$\log I$
20	6640	1.301	3.822
19	6970	1.279	3.843
18	7380	1.255	3.868
16	8240	1.204	3.916
14	9390	1.146	3.973
12	11000	1.079	4.041
10	13100	1.000	4.117

Plot this on a graph; the best fit line is

$$\log I = -0.9816 \log R_c + 5.1000$$



so $\alpha = -0.9824$.

A statistical analysis of the uncertainty in the slope yields $\beta = -0.98 \pm 0.01$.

Graphical fitting of the steepest and shallowest lines yields $\beta = 0.97 \pm 0.02$.

Marking scheme:

Data	vary only R_c	0.05 pts
	$R_e \geq 1$cm	0.05 pts
	$R_c \geq 10R_e$cm	0.05 pts
	$V \geq 100$V	0.05 pts
	table has units	0.05 pts
	R_c distributed evenly	0.05 pts
	$R_{c,\max} \geq 15$cm	0.05 pts
	$R_{c,\min} \geq 10R_e$	0.05 pts
	$R_{c,\min} \leq 10$cm	0.05 pts
	Correct calculations of derived quantities	0.05 pts
	7 or more points	0.30/0.30
	6 points	0.25/0.30
	5 points	0.20/0.30
	4 or fewer points	0.10/0.30
Plotting	covers $> 50\%$ of area	0.10 pts
	Axis labels	0.05 pts
	Axis units correct	0.05 pts
	one plotting mistake	-0.05/-0.10
	two or more plotting mistakes	-0.10/-0.10
Fit	line drawn on graph	0.10 pts
	slope correctly computed with units	0.10 pts
	$-1.03 < \alpha < -0.97$	0.10 pts
	uncertainty of slope computed	0.10 pts
	$\delta\alpha \leq 0.03$	0.10 pts
	sum	**1.5 pts**

B.1: Use all three sets of data, and the exponents from all three, and then average the results

$$\log C = \log I - 1.495 \log V - 0.9854 \log L_e + 0.9781 \log R_c$$

which gives

$$C = (0.0165 \pm 0.0003)\text{mA/V}^{3/2}$$

The theoretical value is approximately:

$$\frac{8\pi\epsilon_0}{9}\sqrt{\frac{2e}{m}} \approx 1.47 \times 10^{-5}\text{A/V}^{3/2}.$$

Note that there is a nasty correction (the texts usually call it β, which is not the same as our exponent), that we use in the code, but aren't expecting students to find, because of this correction, we don't expect the theoretical value to hold. Students who try to solve the theoretical problem will be vexed by this.

For space reasons, we write numerical C below without explicit units, but using the units of $\mu\text{A/V}^{3/2}$, that is

$$C = 16.5\mu\text{A/V}^{3/2}$$

Students *must* have clear units!

Marking scheme:

Theory	clear statement	0.20 pts
Fit	Used $R_c = 10R_e$	0.10 pts
	C computed	0.10 pts
	More than 9 data points	0.20/0.20 pts
	8 or 9 data points	0.15/0.20 pts
	7 or 8 data points	0.10/0.20 pts
	5 or 6 data points	0.05/0.20 pts
	C has correct units	0.10 pts
	$16.2 \leq C \leq 16.8$	0.10/0.10 pts
	$15.9 \leq C \leq 17.1$	0.05/0.10 pts
	uncertainty computed	0.10 pts
	$0.1 < \delta C \leq 0.03$	0.10 pts
	$0 < \delta C \leq 0.05$	0.05/0.10 pts
	sum	**1.0 pts**

Clear statement of theory means that somewhere there is a justification for the data they are collecting and using. This can be in the form of the log formula; words are not necessary. Reusing data is okay.

C.1: Start by assuming that L_e matters, and look at values near R_c. Repeat for other variables. Remember that C depends on the ratio between R_c/R_e, so change these together!

Using nearest half integers, we have for the first equation

$$I_\infty = C\frac{L_e}{R_c}V^{3/2}$$

so that

$$F = \frac{I_{measured}}{C\frac{L_e}{R_c}V^{3/2}}$$

R_c cm	R_e cm	L_e cm	V V	I mA	I_∞ mA	F
10	1	10	1000	535	500	1.071
12	1.2	10	1000	470	416	1.129
8	0.8	10	1000	647	624	1.036
10	1	12	1000	630	599	1.051
10	1	8	1000	451	400	1.129
12	1.2	12	1000	537	500	1.075
8	0.8	8	1000	537	500	1.075
10	1	10	1100	617	576	1.071
10	1	10	900	457	426	1.072

From this we conclude that if $R_c \uparrow$, $F \uparrow$; if $L_e \uparrow$, $F \downarrow$; if $V \uparrow$, F doesn't change.

Also, we notice that the ratio R_c/R_e seems to be the important quantity.

Marking scheme:

Data	clearly collected	0.10 pts
Data	$R_c \uparrow \Longrightarrow F \uparrow$	0.10 pts
	$L_e \uparrow \Longrightarrow F \downarrow$	0.10 pts
	$V_c \uparrow$: F no significant change	0.10 pts
	$R_e \uparrow$: F no significant change	0.10 pts
	sum	**0.5 pts**

C.2: We propose

$$F = A + B\frac{R_c}{L_e}$$

with $x = R_c/L_e$.

Marking scheme:

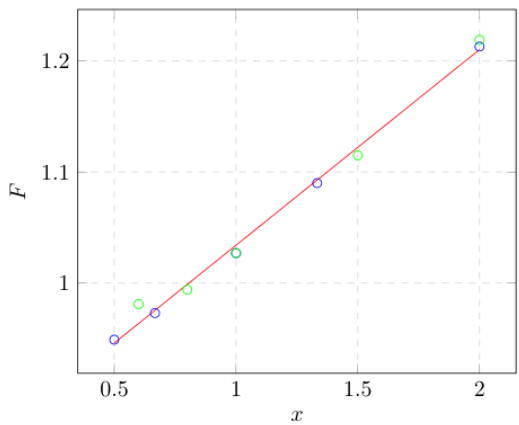
Theory	clear statement	0.20 pts
Def	$x = R_c/L_e$	0.30/0.30 pts
	$x = L_e/R_c$	0.15/0.30 pts
	sum	**0.5 pts**

Any multiple of R_c/L_e is also acceptable.

C.3: It is important to collect data that varies R_c and L_e independently, so as to not bias our hypothesis. We will also keep the ratio with $R_c/R_e = 10$, in order to avoid other effects with the constant in part B.

R_c (cm)	L_e (cm)	I (mA)	I_∞	x	F
20	10	898	740	2.000	1.213
20	15	1210	1110	1.333	1.090
20	20	1520	1480	1.000	1.027
20	30	2160	2221	0.667	0.973
20	40	2810	2961	0.500	0.949
6	10	2420	2467	0.600	0.981
8	10	1840	1850	0.800	0.994
10	10	1520	1480	1.000	1.027
15	10	1100	987	1.500	1.115
20	10	902	740	2.000	1.219

We plot the results below; blue are the values of fixed R_e while green are the values of fixed L_e.



The result is

$$F(x) = 0.8579 + 0.1762x$$

Which is in error at $x = 1$ by about 3%.

If you are thinking that this looks like a quadratic fit might be better, you are correct, but there really isn't time to do that for this experiment.

Marking scheme:

Data	vary L_e	0.10 pts
	vary R_c	0.10 pts
	$R_e \geq 0.5$cm	0.05 pts
	$R_c = 10R_e$cm	0.05 pts
	$V \geq 500$V	0.05 pts
	table has units	0.05 pts
	$L_e \geq 10$cm	0.05 pts
	$L_{e,\max} \leq 40$cm	0.05 pts
	L_e well distributed	0.05 pts
	R_c	0.05 pts
	Correct calculations of derived quantities	0.10 pts
	10 or more points	0.30/0.30
	9 points	0.25/0.30
	8 points	0.20/0.30
	6 or 7 points	0.10/0.30
	5 or fewer points	0.05/0.30
Plotting	covers $> 50\%$ of area	0.10 pts
	Axis labels	0.05 pts
	Axis units correct	0.05 pts
	one plotting mistake	-0.05/-0.10
	two or more plotting mistakes	-0.10/-0.10
Fit	line drawn on graph	0.10 pts
	slope correctly computed with units	0.10 pts
	$0.17 < B < 0.18$	0.10 pts
	sum	**1.5 pts**