Ideal Gas, Vapor Pressure, and Heat Conduction [20 pts.]

In this experimental exam, you will study three physical phenomena: the behavior of an ideal gas, the vapor pressure of water, and heat transfer by conduction. The objective of the exam is to analyze the relationships between pressure P, air volume V, temperature T, and time t, present your results in relevant graphs, and estimate associated physical constants and their associated uncertainties.

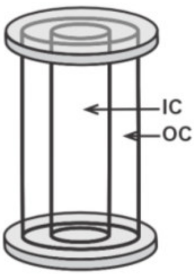


Fig.1a

Fig.1b

Overview of the Experimental Setup

Before beginning the experiment, read the assembly description, list of materials, general instructions, and procedure for each part of the test.

The apparatus consists of two concentric cylinders made of acrylic (Fig. 1a). The inner cylinder (hereafter **IC**) can contain air and liquids, and the air inside **IC** is referred to as confined air (hereafter **CA**). The amount of liquid in the **IC** can be controlled by connecting the syringe to **hose D**.
The outer cylinder (hereafter **OC**) can only be filled with water and acts as a thermal jacket. Its water level can be adjusted using inlet and outlet hoses and valves, while its temperature is controlled by an electric heater. Two sensors are installed inside the **IC**: one for pressure and one for temperature. A separate temperature sensor is available in the **OC**.

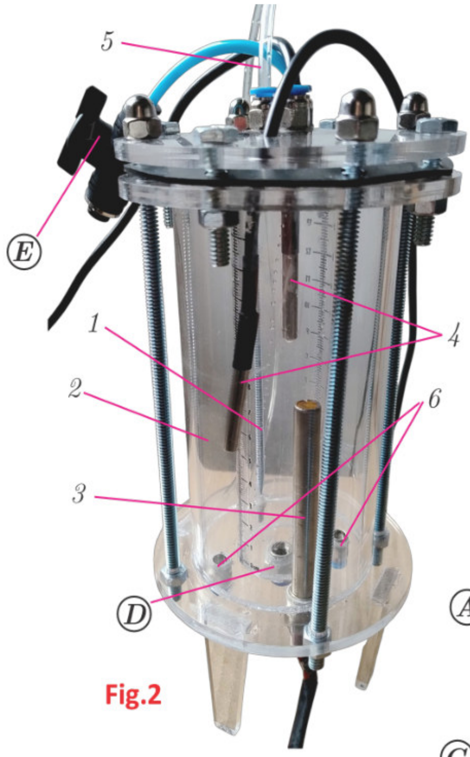


Fig.2

Fig.3

Fig.4

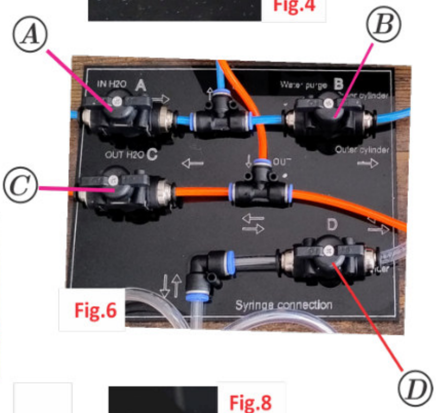


Fig.6

Fig.5

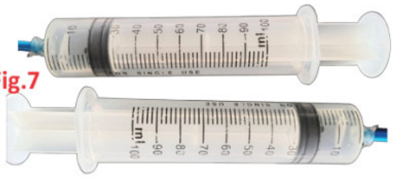


Fig.7

Fig.8

Experimental Equipment

Main Apparatus (see Fig. 2)

1. **Inner cylinder** (**IC**): Graduated with two centimeter scales. Its fixed lids keep the contents airtight.

2. **Outer cylinder** (**OC**): Acts as a thermal jacket.

3. **Heater:** Installed in the **OC** and powered by the electronic module.

4. **Temperature sensors**: Two sensors, connected to the electronic module by black wires, measure the temperature in each cylinder.

5. **Pressure sensor hose:** Connects **CA** to a pressure sensor hosted in the electronic module, allowing measurement of the internal pressure.

6. **OC water connections**: Two hoses provide water with an inlet to and outlet from the **OC.**

Valve E — IC atmospheric connection: Opens or closes the connection between the **IC** and the atmosphere.

Electronic Module (see Fig. 3)

The electronic module must be connected to the main power before use. The power switch, labeled **Power**, is located at the back of the module (Fig. 4).

The digital display provides the following readings:

- **Internal pressure:** Pressure inside the **IC** (**INT. PRESSURE**).
- **Atmospheric pressure:** Local atmospheric pressure (**ATM. PRESSURE**).
- **Internal temperature:** Temperature inside the **IC** (**INT. TEMPERATURE**).
- **External temperature:** Temperature inside the **OC** (**EXT. TEMPERATURE**).
- **Stopwatch:** Timer reading (**TIMER**).

Four numbered buttons below the screen control the main module functions:

- **Button 1 — Timer**: Starts or stops the stopwatch.
- **Button 2 — Reset**: Clears the timer reading.
- **Button 3 — Graph display**: toggles between Graph display and numerical display. **This function is not used in the experiments.**
- **Button 4 — Heater**: Turns the heater on or off. When the heater is active, the animated indicator in the lower-right corner of the screen alternates colors between red and green.

Note: Never turn on the heater if it is not fully immersed in water. When the OC temperature reaches 65 °C, the heater is deactivated. It may be reactivated only after the animated indicator is no longer red.

The right side of the module (Fig. 5) contains the connections for the **IC** temperature sensor (Internal Temp.), the **OC** temperature sensor (**External Temperature**), the heater (**Water Heater**), and the hose that connects **CA** to the pressure sensor (**Internal Pressure**)

Hydraulic Module (see Fig. 6)

The hydraulic module contains four valves, labeled **A**, **B**, **C**, and **D**. Each valve controls a specific fluid pathway in the apparatus:

- **Valve A — Water inlet to OC**: Controls the flow of water into the **OC** through **hose A** (**IN H2O**).
- **Valve B — Recirculator valve:** Controls the flow of water through the recirculation cycle, described in the pump operation instructions below.
- **Valve C — Water outlet from OC:** Controls the flow of water out of the **OC** through **hose C** (**OUT H2O**).
- **Valve D — Syringe connection:** Controls the inlet and outlet of fluids in the **IC** using the syringe connected to **hose D** (Fig. 2). If the valve is leaky you can detach the hose from the setup and connect the syringe directly to the hose.

Pump and Valve Operation

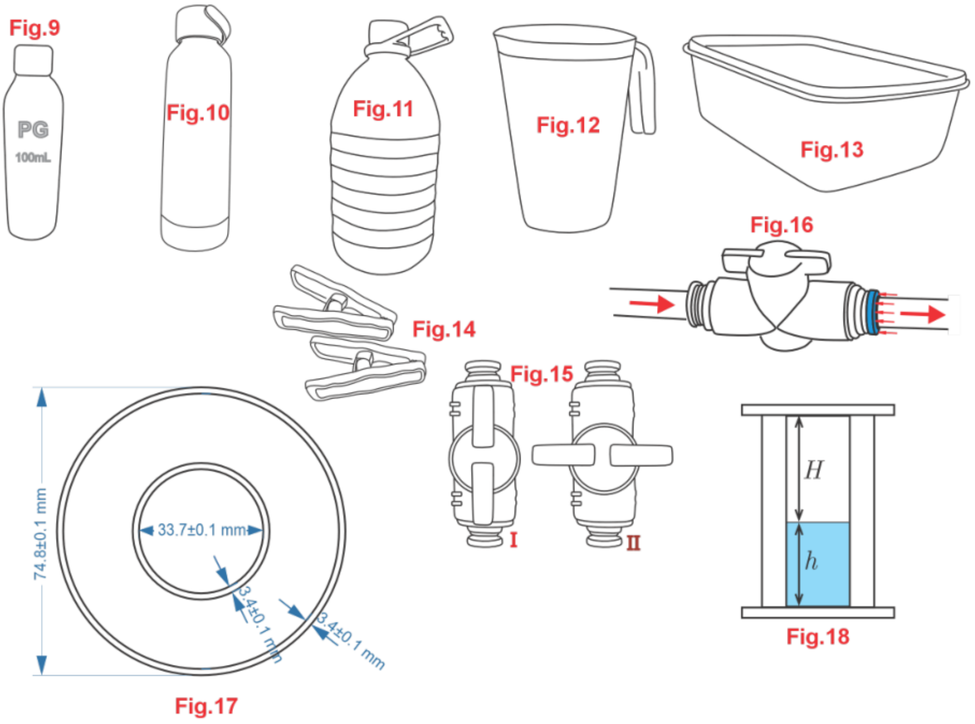
The pump switch (**PUMP**), located on the front of the electronic module (Fig. 8), activates the pump inside the electronic module. Before turning on the pump, set the valves according to the operation you want to perform. Use the clips to secure the hoses to the edge of the containers before operating the pump.

- **Fill the OC with water:** Open **valve A** and close **valves B** and **C**. Place **hose A** in a water container. When the desired level is reached, close **valve A**. Make sure the water level remains at least 1 cm below the cover of the cilinder. *If water does not enter **OC** efficiently, immediately open and close **valve B** to purge the pump.*
- **Drain water from the OC:** Close **valve A** and open **valves B** and **C**. Place **hose C** in a container to collect the water.
- **Homogenize the OC temperature:** Open **valve B** and close **valves A** and **C**. In this configuration, water circulates through the **OC** in a closed loop, helping to make the temperature uniform.

For the following operations, connect the syringe to **hose D**. The pump is not used in these cases.

- **Add liquid to the IC:** Open **valves D** and **E,** then use the syringe to introduce the liquid.

- **Remove liquid from the IC:** Open **valves D** and **E**, then use the syringe to draw out the liquid.



Additional Items: the following materials are supplied for the experiment.

- **Two 100 mL syringes** with a connecting hose (Fig. 7). One syringe is for water, the other for **PG**.

- **Propylene glycol (PG):** A container with 100 mL of **PG**, whose vapor pressure is negligible over the temperature range used in this experiment (Fig. 9). **Warning: PG is miscible with water.** It can also hydrate if open to the air. It is recommended that you keep the container closed while not using it.

- **Cold water:** A thermal container with 500 mL of cold water (Fig. 10).

- **Room-temperature water:** A bottle of water at room temperature (Fig. 11).

- **Pouring container:** A container used for pouring water (Fig. 12).

- **Excess-water jug:** A jug for collecting excess water (Fig. 13).

- **Clips:** For securing hoses and supporting components during the experiment (Fig. 14).

- **Paper towels** for cleanup.

Important Operating Notes

Keep the following notes in mind when setting up and operating the apparatus:

- **Valve positions:** Each valve has only two positions: open (**I**) and closed (**II**) (Fig. 15).

- **Hose connection:** To remove a hose, push the hoop and pull the hose out. To insert a hose, push it fully into place (Fig. 16). **Note that the apparatus is fully assembled at the start of the exam and the hoses normally do not need to be reconnected in the usual operation of the apparatus.**

- **Cylinder dimensions:** Figure 17 shows the cross-sectional dimensions of the cylinders.

- **Notation:** Figure 18 defines the notation used for the **IC** contents: $\boldsymbol{h}$ is the height of the liquid, and $\boldsymbol{H}$ is the height of the air above the liquid.

- **Volume corrections:** The internal volume of the connecting hoses is approximately compensated by the volume of the temperature sensor inside the **IC**. Therefore, these effects should be neglected.

Constants and Reference Values

The following constants and reference values are used throughout the experiment:

- **Avogadro constant:** $N_A = 6.022 \times 10^{23}\,\mathrm{mol}^{-1}$
- **Specific heat capacity of water:** $c_0 = 4.184 \times 10^3\,\mathrm{J/(kg{\cdot}K)}$
- **Density of water:** $\rho_0 = 1000\,\mathrm{kg/m^3}$
- **Molar mass of water:** $\mathrm{M}_0 = 18.02\,\mathrm{g/mol}$
- **Molar mass of air:** $\mathrm{M}_a = 28.96\,\mathrm{g/mol}$
- **Molar volume of an ideal gas at normal conditions:** $V_m = 22.4\,\mathrm{L/mol}$, for $T_0 = 273.15\,\mathrm{K}$ and $P_0 = 101.3\,\mathrm{kPa}$

Part A [4.0 pt]. Isochoric (isovolumetric) process of an ideal gas

CA obeys the ideal gas equation of state,

$$PV = nRT \tag{1}$$

where R is the universal gas constant.

Procedure

Introduce **PG** into **IC** to $h = 4.5\,\mathrm{cm}$ and close valves **D** and **E.** This ensures that the volume of **CA** is fixed.

The density of ambient air in Bucaramanga varies with local temperature and pressure. For the following question, use the time-averaged value $\rho = 1.12\,\mathrm{kg/m^3}$.

A1. [0.4 pt] Determine the mass m, number of moles n, and the total number of air molecules N of the **CA**.

1. Fill **OC** with room temperature water close to the top.

2. Turn on the heater. Don't forget to homogenize the temperature by using the pump.

A2. [1 pt] Record in a table the pressure P of the CA as a function of its temperature T.

A3. [0.9pt] From the data obtained in question A2, plot the behavior of pressure as a function of temperature.

Under IPhO competitive conditions, the sensors may be intentionally de-calibrated to alter the expected value of R with respect to the reference value.

A4. [1.0pt] Use the graph from A.3 to determine the experimental value of R.

The thermal pressure coefficient at constant volume is defined by the relationship:

$$\beta_0 = \tfrac{1}{P_0}\left(\tfrac{\Delta P}{\Delta T}\right), \tag{2}$$

where P_0 is the pressure of the system at the reference temperature T_0 as indicated in the reference constants and values.

A5. [0.7pt] Determine the value of the coefficient β_0 for air.

Remove the PG from the IC and clean it before proceeding to the other parts. If you proceed to part B, do not remove the hot water from OC.

Part B [8.0 pt]. Vapor Pressure

In a closed container containing a liquid and its vapor at equilibrium, the vapor pressure depends on the absolute temperature T according to the Clausius–Clapeyron equation:

$$P_v = P_{v0} e^{-\frac{Q_v}{R}\left(\frac{1}{T}-\frac{1}{T_0}\right)} \tag{3}$$

Here, T_0 is the reference temperature, taken as 0 °C. P_{v0} is the vapor pressure at T_0. R is the universal gas constant, and Q_v is the molar latent heat of vaporization of water.

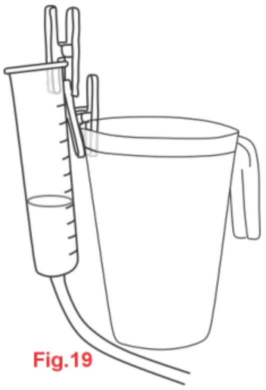


Fig.19

Procedure

Remember that PG and water are miscible. Use different syringes for water and PG

- Fill the **OC** with water until the water level is close to the top.

- Fill a syringe with water, connect it to **IC** and open valve **E**, remove the syringe plunger, and raise the syringe until it reaches the water level at $h = 5.0\,\text{cm}$. Close valve **E** and then secure the syringe to the jug as shown in Figure 19. This arrangement keeps the pressure inside the **IC** approximately equal to the atmospheric pressure. Make sure no air bubbles remain in the hose.

- Heat the water in the **OC** to 65 °C. Remember to use the recirculator system to homogenize the water temperature.

- Wait until the temperature starts to decrease to start recording H as a function of T.

- To lower the temperature further, you may replace the water in the **OC** with the supplied cold water at the appropriate time.

B1. [1.0 pt] Record the height H as a function of temperature T in a table

B2. [1.0 pt] Use the data obtained in B1 to plot H as a function of T.

The vapor pressure is relatively small around room temperature and the $H(T)$ curve may be approximated as linear.

B3. [0.5 pt] Use the graph from question B2 to extrapolate and find the value of H_0 (H at 0 °C).

B4. [1.0 pt] Assuming that **IC** contains both dry air plus water vapor, deduce an algebraic expression for the vapor pressure P_v in terms of P_{atm} *(atmospheric pressure)*, H_0, H, T_0 and T. For the purposes of this problem, you can assume that the vapor pressure at 0 °C is zero.

For the following questions, use the reference value $R = 8.31\,\text{J/(mol·K)}$.

B5. [4.0 pt] Using equation (3) and the expression from question B4, construct an appropriate graph to determine the experimental value of Q_v.

B6. [0.5 pt] From the value of Q_v, determine L_v,*(the latent heat of vaporization per unit mass)* indicating the formula that you used.

Part C [8.0 pt]. Heat Conduction

In this part of the exam, you will study the heat exchange between the water in the **OC** and the water in the **IC** through the wall that separates them. By analyzing the time evolution of the water temperature in the **IC**, T_{IC}, and the water temperature in the **OC**, T_{OC}, the objective of this section is to estimate the effective thermal resistance of the wall, the equilibrium temperature of the system, and the thermal conductivity of acrylic.

Heat conduction through a solid wall follows the relation:

$$\frac{\Delta Q}{\Delta t} = \frac{1}{R_{Th}}(T_{OC} - T_{IC}) \tag{4}$$

where ΔQ is the heat received by the water in the **IC** through the wall during the time interval Δt. The effective thermal resistance R_{Th} depends on the material and geometry of the wall separating **IC** and **OC**.

Procedure

1. Set the water level in the **OC** to $h = 15\,\text{cm}$.

2. Heat the water in the **OC** to 65 °C. Use the pump to homogenize the water temperature.

3. Set the water level in the **IC** to $h = 10\,\text{cm}$ and start the stopwatch.

C1. [1.0 pt] Record the internal temperature T_{IC} and external temperature T_{OC} as a function of time t

C2. [1.0 pt] On a single graph, plot T_{IC} and T_{OC} as functions of time t.

For questions C3 and C4, ignore the heat capacity of the apparatus. $\mathrm{T_{eq}}$ refers to the equilibrium temperature that would be reached by both IC and OC if there were no heat transfer to the environment.

C3. [0.7 pt] Plot $\mathrm{T_{OC}}$ - $\mathrm{T_{IC}}$ as functions of both $\mathrm{T_{OC}}$ and $\mathrm{T_{IC}}$ on the same graph with $\mathrm{T_{OC}}$ and $\mathrm{T_{IC}}$ together on the same temperature axis and $\mathrm{T_{OC}}$ - $\mathrm{T_{IC}}$ on the other axis.

C4. [1.1 pt] From the graphs constructed in C3, determine T_{eq}. It is not necessary to calculate uncertainty for this question.

If the variables are measured a finite number of times, Equation (4) can be expressed as

$$\frac{T_{IC,j} - T_{IC,(j-1)}}{t_j - t_{j-1}} \propto \bar{T}_{OC} - \bar{T}_{IC} \tag{5}$$

where j represents the measurement number and $\overline{T}$ are the average temperatures at each cylinder during the interval t_{j-1} to t_j

C5. [1.0 pt] Graph the expression on the left hand side of Equation 5 as a function of the expression on the right hand side

C6. [1.6 pt] Determine the value of R_{Th} from the graph constructed in C5.

Fourier's law for radial heat conduction through a slim cylindrical wall can be written as:

$$\frac{dQ}{dt} = -\lambda A \frac{dT}{dr} \tag{6}$$

where A is the area of the wall, λ is the thermal conductivity of the wall material, and r denotes the distance from the axis of the cylinder.

C7. [1.6 pt] Using equations (4) and (6), determine the value λ of the thermal conductivity of acrylic (the material separating IC and OC), indicating the formula that you used.