6. Charged particle (12 points)

A particle of mass m and charge q is in a homogeneous magnetic field with induction B (the vector is parallel to the z-axis). The characteristic time of the system is the cyclotron period of the particle $T_B = 2\pi m/Bq$. The system is situated in between two parallel electrodes, which can be used to create an homogeneous, parallel to the x-axis electric field E.

1) The particle is at rest. At the moment of time $t = 0$, the electric field E is switched on; after a short time interval τ ($\tau \ll T_B$), it is switched off, again. What will be the trajectory of the particle?

2) Let p_x and p_y denote the x- and y components of the momentum of the particle. Sketch the trajectory of the particle in (p_x, p_y)-plane and depict the vectors of the momentum for the moments of time $t_n = nT_B/4$ ($n = 1, 2, 3$ and 4).

3) Consider the situation when the on-off switching of the electric field is done periodically, starting with $t = 0$, after equal time intervals $\Delta t = T_B/4$. Sketch the particle trajectories in (p_x, p_y)- and (x, y)-planes.

4) Let the period be short, $\Delta t \ll T_B$ (but still much longer than the duration of pulse, $\Delta t \gg \tau$). Show that after the n-th pulse (at the time moment $t_n = n\Delta t$), the momentum of the particle can be represented as the sum of n vectors $\vec{p}_i$, where all the component-vectors are equal in modulus (the modulus being independent of n), and the neighboring vectors ($\vec{p}_i$ and $\vec{p}_{i+1}$, $i = 1, 2, \ldots$) have equal angles between them.

5) Consider the limit case $\Delta t \to 0$, so that $E\tau/\Delta t \to E_k$ (E_k denotes the time-average of the elctric field). Sketch the particles trajectory in (p_x, p_y)-plane and express the particles mean velocity (vectorially; averaged over the cyclotron period) via the quantities E_k and B.

6) Let us return to the non-zero (but still small, $\Delta t \ll T_B$) time-intervals. Let us consider the case when the pulses are of variable polarity: for the $2n$-th pulse, the electric field is $+E$, and for the $2n+1$st pulse $-E$. Find the particles average velocity, (vectorially) averaged over the cyclotron period.