7. Telescope (12 points)

As it is well known, a telescope makes it possible to see the stars in daylight. Let us study the problem in more details. Consider a simplified model of the eye: a single lens with focal length $f = 4\,\text{cm}$ and diameter $d = 3\,\text{mm}$ creating an image on screen (retina). The model of a telescope is similar: a lens of focal length $F = 2\,\text{m}$ and diameter $D = 20\,\text{cm}$ creating an image in focal plane (where eg. a film can be put). In your calculations, the following quantities can be used: the density of the light energy radiated from a unit Solar surface in unit time w_0 (the light power surface density); the ratio of the star and Sun distances $q = 4 \cdot 10^5$ (we assume that the star is identical to the Sun); Solar angular diameter $\phi \approx 9\,\text{mrad}$. *Remark:* If the answer contains w_0 then numerical answer is not required.

1) Consider a sheet of paper, the normal of which is directed towards the Sun. What is the surface density of the light power w_1 arriving to the sheet from the Sun?

2) Find the net power P_2 of the light, which is focused by the telescope into the image of the star.

3) Assume that blue sky is as bright as a sheet of gray paper illuminated by Sun. You may assume that in the direction, perpendicular to the sheet, the ratio of the light power scattered by the paper into a 1-steradian space angle, to the net light power arriving to the sheet, is $\alpha \approx 0{,}1$ (this corresponds to the dissipation of ca 70 % light energy in the gray paper). What is the surface density of the light power in the focal plane of the telescope w_3, due to the blue sky?

4) While studying the star image, let us ignore all the effects other than diffraction. Estimate the surface density of the light power in the center of the star image w_2 (in the focal plane of the telescope), due to the light arriving from the star.

5) Provide an expression for the ratio of the surface densities of the light powers k in the middle of the star image, and in a point farther away from it.

6) Is it possible to see a star in daylight using a telescope? Plain eye? Motivate yourself.