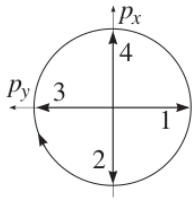
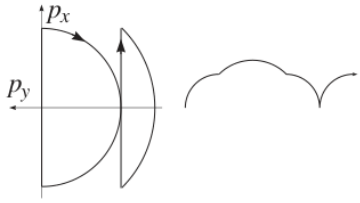
6. Charged particle (12 pts)

1) The particle acquires the velocity $v = Eq\tau/m$ and starts moving along a circle of radius R, with $mv^2/R = Bvq$, hence $R = E\tau/B$.



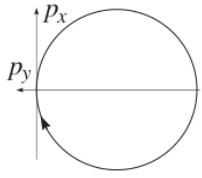
2)



3)

4) Let us consider the vectorial sum of the momenta given to the particle in different moments of time. During the time interval Δt, all the component-vectors are rotated by the angle $2\pi\Delta t/T_B = \tau Bq/m$. Thus, with each impulse, a vector $\vec{P}$ with modulus $P = Eq\tau$ is added; the angle between the lastly added vector, and the previously added vector is $\alpha = \Delta t Bq/m$.

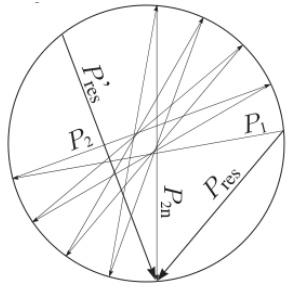
5) All these vectors, when added according to the triangle rule, form a circle of radius $\mathcal{R} = P/\sin\alpha \to P/\alpha = E_k m/B$.



Hence, the average velocity $v_y = -\mathcal{R}/m = E_k/B$, $v_x = 0$.

6) Two subsequent momenta along x-axes result in net moment along y-axes $\mathcal{P}_y = P\alpha$. The sequence of such moment pairs form a (nearly) circle (actually, regular equilateral polygon), composed of vectors (with modulus $\mathcal{P}_y$), the angle between of which is 2α (see Fig.). The radius of the circle is $\mathcal{P}_y/2\alpha = P/2 = \frac{1}{2}Eq\tau$, and its center coordinates are $mv_x = -Eq\tau/2$, $mv_y = 0$. After an even number of impulses, the endpoint of the particles momentum lies on that circle. Thus, averaged over the moments of time $2n\Delta t$, the average velocity is $v_x = -Eq\tau/2m$.

For odd number of impulses, one has to add the lastly given momentum $\vec{P} = (Eq\tau, 0)$; hence, a similar circle is formed, except that the center is shifted by $\vec{P}$: the center coordinates are $mv_x = Eq\tau/2$, $mv_y = 0$. Correspondingly, averaged over the moments of time $2n\Delta t$, the average velocity is $v_x = +Eq\tau/2m$. Averaged over all the moments of time, the final result is $v_x = v_y = 0$.



The figure represents the net moment P_{res} after $2n$-th impulse, and also the net impulse P'_{res} for another time moment $2n'\Delta t$. For an odd number of impulses, the pattern is exactly the same, except that all the vectors have opposite direction (because the lastly added component, the vertical vector, has opposite direction).