7. Telescope (12 pts)

1) The light flux density decreases inversely proportionally to the square of the distance, therefore $w_1 = w_0 R_p^2/L_p^2$, where R_p is the solar radius, and L_p — the solar distance. Due to $\phi = 2R_p/L_p$, we obtain $w_1 = w_0\phi^2/4$.

2) The previous result can be applied to the star flux density, which is $q^{-2}w_1$; hence $P_2 = \frac{1}{4}\pi D^2 w_1 q^{-2} = w_0\pi(\phi D/4q)^2$.

3) The paper surface area S radiates towards the lens of the telescope the power $P_3 = w_1\alpha S(\frac{\pi}{4}D^2/L^2)$, where L is the telescope distance. The image of this piece of paper has size $s = SF^2/L^2$; thus, $w_3 = P_3/s = w_1\alpha(\frac{\pi}{4}D^2/F^2) = w_0\alpha\pi(\phi D/4F)^2$.

4) The angular distance of the first diffraction minimum (using the single slit approximation — circle is actually not a slit) is λ/D. Hence, the bright circle radius can be estimated as $\delta = F\lambda/D$. Consequently, $w_2 = P_2/\pi\delta^2 = w_0(\phi D^2/4qF\lambda)^2$.

5) $k = (w_2 + w_3)/w_3 = 1 + (\alpha\pi)^{-1}(D/\lambda q)^2 \approx 4$ (assuming $\lambda \approx 500\,\mathrm{nm}$).

6) $k-1 \sim 1$ (or $k-1 > 1$) means that the star can be easily seen (as is the case for the telescope); $k-1 \ll 1$ means that the star cannot be seen (for the eye, $k-1 \approx 1\cdot 10^{-4}$).