1. Rubber fiber (12 pts)

Fibers made of elastic rubber can be stretched to lengths l, much longer than the length in undeformed state l_0. For such rubbers, the net volume of the fiber remains constant.

1) Express the cross-sectional area S of such a fiber in a deformed state via its length l and initial dimensions l_0, S_0 (1 pt).

2) For small deformations of an elastic material, the stretching force F and deformation x are related to each other by the Hooke's law $F = k_0 x$, where the stiffness $k_0 = E_0 S_0 / l_0$ and E_0 is the Young's modulus of the rubber. For non-small (possibly large, $l \gg l_0$) deformations of elastic rubber, however, the Hooke's law is substituted by a non-linear law, $F(l) = a + \frac{b}{l}$ (breaking of this law at very large values of l will not be studied here). Express the constants a and b in terms of l_0, S_0, and E_0 (2 pt).

3) Suppose such a fiber is stretched by some force up to the length l. A small change ΔF of the stretching force results in a small change in the length $\Delta l \ll l$. Express ΔF in terms of l, l_0, S_0, E_0, and Δl (1 pt).

4) Suppose a small body is fixed to an one end of the fiber and the system is put into rotation around the other end of the fiber. In the case of a circular motion of the body, express the length of the fiber l via l_0, S_0, E_0, and the kinetic energy of the body K (the kinetic energy of the fiber and gravity can be neglected). (1.5 pts)

5) Let us analyse a slightly non-circular motion of the body. Let us describe the motion of the system by the length change of the fiber $r(t) = l(t) - l(0)$, the radial $v_r(t)$ and tangential $v_t(t)$ velocities of the body (the components respectively parallel and perpendicular to the fiber). The initial values of these quantities are designated as $L \equiv l(0)$, $V_r \equiv v_r(0)$, and $V_t \equiv v_t(0)$. The values L and V_t are chosen so that if the initial radial velocity were zero, the motion would be circular. Write down two independent equations relating $r(t)$, $v_r(t)$, and $v_t(t)$ to each other (using also the mass of the body m, together with the parameters L, V_r, V_t, l_0, S_0, E_0). (3.5 pts).

6) Find the relationship between $r(t)$ and $v_r(t)$ (containing also the parameters m, L, V_r, V_t, l_0, S_0, E_0) assuming that $|r| \ll L$, and find the period T of small oscillations of $r(t)$. Simplify the expression of T for $L \gg l_0$ (3 pts).