2. Planets (6 pts)

1) First method: determine the tangents to the graph at the points where the curve crosses the horizontal axis, $a_1 \approx -2.8$ and $a_2 \approx 16$ (in graph units), respectively. Then, $a_2 = \omega\frac{-k}{1+k}$ and $a_2 = \omega\frac{k}{1-k}$. The graph units will cancel out from the ratio of these to tangents, $\varepsilon = -\frac{a_2}{a_1} = \frac{1+k}{1-k} \approx 5.7$, hence $k = \frac{1-\varepsilon}{1+\varepsilon} \approx 1.4$.

Second method (more precise): determine the distances between neighbouring minimum and maximum, $t_1 \approx 1.2$, and between neighbouring minima $t_2 \approx 4.6$ (in graph units), respectively. $\mu = \frac{t_1}{t_2} = \frac{\pi - 2\arcsin k}{2\pi} \approx 0.261$, hence $k = \sin[(\frac{1}{2} - \mu)\pi] \approx 1.47 \approx 1.5$.

2) For the maximal angular displacement φ_m, $\sin\varphi_m = k = \sin[(\frac{1}{2} - \mu)\pi]$, hence $\varphi_m = (\frac{1}{2} - \mu)\pi = 0.75\,\mathrm{rad} \approx 3.6$ units. Therefore, the unit is $\varphi_m/(\frac{1}{2} - \mu)\pi \approx 4.8$.

3) If the angular velocities of the planets are ω_1 and ω_2, the seeming angular velocity (as seen from the system, where both star and the observer planet are at rest) is $\omega = \omega_1 - \omega_2$. From the Newton II law, $GMr_i^{-2} = \omega_i^2 r_i$, where $i = 1, 2$ and r_i is the planet's orbital radius. So, $\omega_i = \sqrt{GMr_i^{-3}}$ and $\omega = \sqrt{GM}(r_2^{-1.5} - r_1^{-1.5}) = \sqrt{GM}r_1^{-1.5}(1 - k^{-1.5})$. Finally, the square of the observed period $T^2 = (2\pi/\omega)^2 = 4\pi^2 r_1^3/GM(1 - k^{-1.5})^2$ and $r_1 = [T^2 GM(1 - k^{-1.5})^2/4\pi^2]^{1/3}$. Using $T \approx 4.6$ years $\approx 1.45 \cdot 10^8$ s, we arrive at $r_1 \approx 2.5 \cdot 10^{11}$ m; correspondingly, $r_2 = kr_1 \approx 3.7 \cdot 10^{11}$ m.