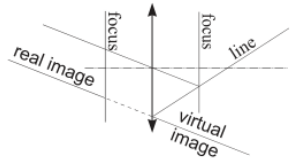
3. Tilt-shift lens (6 pts)

From $f^{-1} = x^{-1} - x'^{-1}$ we obtain $x = \frac{x'f}{x'+f}$. Since the ray passing through the centre of a lens without refraction, from similar triangles we obtain the relationship between the y-coordinates of the image: $y = y'\frac{x}{x'} = \frac{y'f}{x'+f}$. Substituting into $y = ax + b$ we result in $\frac{y'f}{x'+f} = a\frac{x'f}{x'+f} + b$, hence $y' = ax' + b\left(\frac{x'}{f} + 1\right) = x'\left(a + \frac{b}{f}\right) + b$, which defines also a straight line.

1) First we notice that the line, its image, and lens plane intersect in one point, because the image of that point of the line which lays at the lens plane, coincides with itself. Now, it is easy to construct the image, see the figure.



2) First we notice that the distance of the image of the far end of the field (let us designate it by A) from the focal plane equals to the focal length f. So, the lens plane must touch the circle of radius f, drawn around A, see figure. Next we notice that there is one such ray connecting far end of the field and its image, which does not refract — the one passing through the enter of the lens, see figure.

