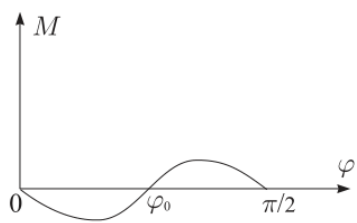
5. 4th order ellipse (6 pts)

1) There are trivial positions $\varphi = 0$ and $\varphi = \pi/2$. Besides, there is a position between these two. At the equilibrium, the vector from the origin to the touching point $\vec{r} = (x, y)$ has to be perpendicular to the tangent at that point. In order to find the tangent, let us differentiate the ellipse formula: $4\frac{x^3}{a^4}dx + 4\frac{y^3}{b^4}dy = 0$, hence, with $dx = 1$, $dy = -\frac{x^3}{y^3}\frac{b^4}{a^4}$, a tangent vector is $\vec{\tau} = \left[1, -\left(\frac{x}{y}\right)^3\left(\frac{b}{a}\right)^4\right]$. The vectors are perpendicular, if the scalar product is zero: $x - y\left(\frac{x}{y}\right)^3\left(\frac{b}{a}\right)^4$, i.e. $\frac{y}{x} = \left(\frac{b}{a}\right)^2 = \varphi = \arctan\frac{y}{x} = \left(\frac{b}{a}\right)^2$.

2) Around each zero φ changes sign. At $\varphi = 0$, small increase in φ will result in a torque trying to return to the initial position, i.e. the torque becomes negative. So, the graph looks like the one below.



3) If the derivative of the graph at equilibrium point is negative, the position is stable; otherwise it is unstable. $\varphi = 0$ and $\varphi = \pi/2$ are stable, $\varphi = \left(\frac{b}{a}\right)^2$ is unstable.